CCuantuMM: Cycle-Consistent Quantum-Hybrid Matching of Multiple Shapes

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Overview and Contributions

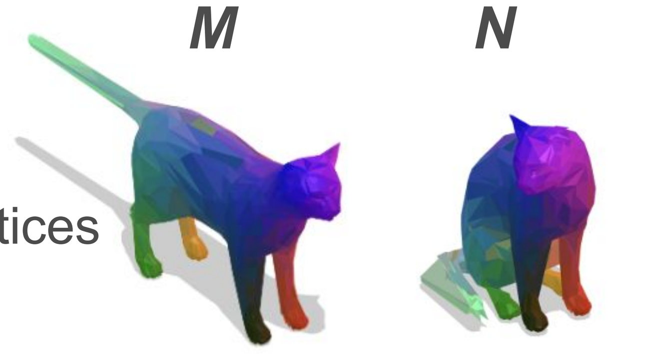
Multi-matching: finding cycle-consistent point-wise correspondences between several shapes

Previous quantum work [1] successfully matches two shapes. We propose a new method supporting multiple shapes. It outperforms AQC-SotA and is on par with classical SotA.

Contributions

1. CCuantuMM: Quantum-hybrid multi-matching algorithm with cycle consistency, using adiabatic quantum computing (AQC)
2. New policy to match three shapes on AQC
3. Policy for extending the three-shape case to *N* shapes

Background



Quadratic Assignment Problem:

Input: Shape *M* and *N*, each discretized into *n* vertices

$$\min_{x\in\mathbb{P}_n} E(x) = x^T W x \qquad x = \mathrm{vec}(X),\quad \mathbb{P}_n \subset \{0,1\}^{n^2},\quad W \in \mathbb{R}^{n^2\times n^2}$$

Output: Optimal permutation *P*

Quantum Annealing: A heuristic that provides a solution to Quadratic Unconstrained Binary Optimization (QUBO) problems:

$$\min_{x\in\{0,1\}^k} x^T Q x \quad \text{where} \quad Q \in \mathbb{R}^{k\times k}$$

Formulating the Three-Shape Case

a) Energy formulation: $\min_{\alpha,\beta\in\{0,1\}^k} = E_{\mathcal{XY}}(P_{\mathcal{XY}}(\alpha)) + E_{\mathcal{YZ}}(P_{\mathcal{YZ}}(\beta)) + E_{\mathcal{XZ}}(P_{\mathcal{XZ}}(\alpha,\beta))$

b) Parameterizing updates to permutation *P* with cyclic alpha expansion [1]: $P(\alpha) = P + \sum_{i=1}^{k} \alpha_i (c_i - I) P$

c) Enforcing cycle consistency: $P_{\mathcal{XZ}}(\alpha,\beta) = (P_{\mathcal{XY}} + \sum_{i=1}^{k} \alpha_i (c_i - I) P_{\mathcal{XY}}) \cdot (P_{\mathcal{YZ}} + \sum_{j=1}^{k} \beta_j (\tilde{c}_j - I) P_{\mathcal{YZ}})$

Problem: This energy is not a QUBO because cycle consistency leads to cubic and bi-cubic terms

Solution: Eliminate these higher-order terms since they have little impact on the solution. This yields a QUBO, *i.e.* is compatible with AQC.

d) Final energy formulation:

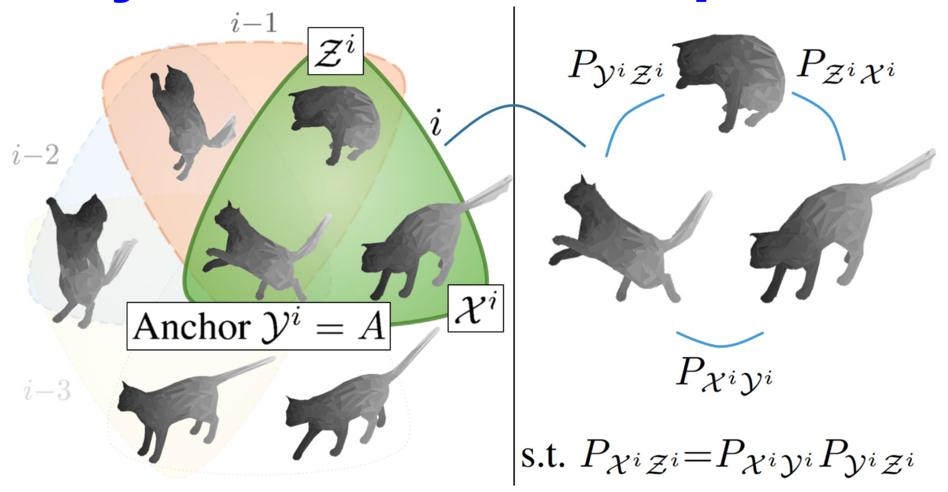
$$\begin{aligned}\min_{\alpha,\beta} \; & \sum_{i=1}^{k} \alpha_i \Big(F_{\mathcal{XY}}(P_{\mathcal{XY}}, C_i) + F_{\mathcal{XZ}}(P_{\mathcal{XZ}}, C_i P_{\mathcal{YZ}}) \Big) + \sum_{j=1}^{k} \beta_j \Big(F_{\mathcal{YZ}}(P_{\mathcal{YZ}}, \tilde{C}_j) + F_{\mathcal{XZ}}(P_{\mathcal{XZ}}, P_{\mathcal{XY}} \tilde{C}_j) \Big) \\ & + \sum_{i=1}^{k}\sum_{l=1}^{k} \alpha_i \alpha_l \Big(E_{\mathcal{XY}}(C_i, C_l) + E_{\mathcal{XZ}}(C_i P_{\mathcal{YZ}}, C_l P_{\mathcal{YZ}}) \Big) + \sum_{j=1}^{k}\sum_{l=1}^{k} \beta_j \beta_l \Big(E_{\mathcal{YZ}}(\tilde{C}_j, \tilde{C}_l) + E_{\mathcal{XZ}}(P_{\mathcal{XY}}\tilde{C}_j, P_{\mathcal{XY}}\tilde{C}_l) \Big) \\ & + \sum_{i=1}^{k}\sum_{j=1}^{k} \alpha_i \beta_j \Big(F_{\mathcal{XZ}}(P_{\mathcal{XY}}\tilde{C}_j, C_i P_{\mathcal{YZ}}) + F_{\mathcal{XZ}}(K_{ij}, P_{\mathcal{XZ}}) + F_{\mathcal{XZ}}(K_{ij}, P_{\mathcal{XY}}\tilde{C}_j) + F_{\mathcal{XZ}}(K_{ij}, C_i P_{\mathcal{YZ}}) + E_{\mathcal{XZ}}(K_{ij}, K_{ij}) \Big),\end{aligned}$$

$$F_{\mathcal{IJ}}(A,B) = E_{\mathcal{IJ}}(A,B) + E_{\mathcal{IJ}}(B,A) \quad \text{and} \quad K_{ij} = C_i \tilde{C}_j$$

This optimisation problem is given to a **quantum annealer**, which outputs a solution. This is repeated for several iterations until convergence.

Extending to an Arbitrary Number of Shapes

We choose an anchor. Then, each shape triplet in the shape collection is optimised using the three-shape formulation with AQC. This is repeated until convergence.

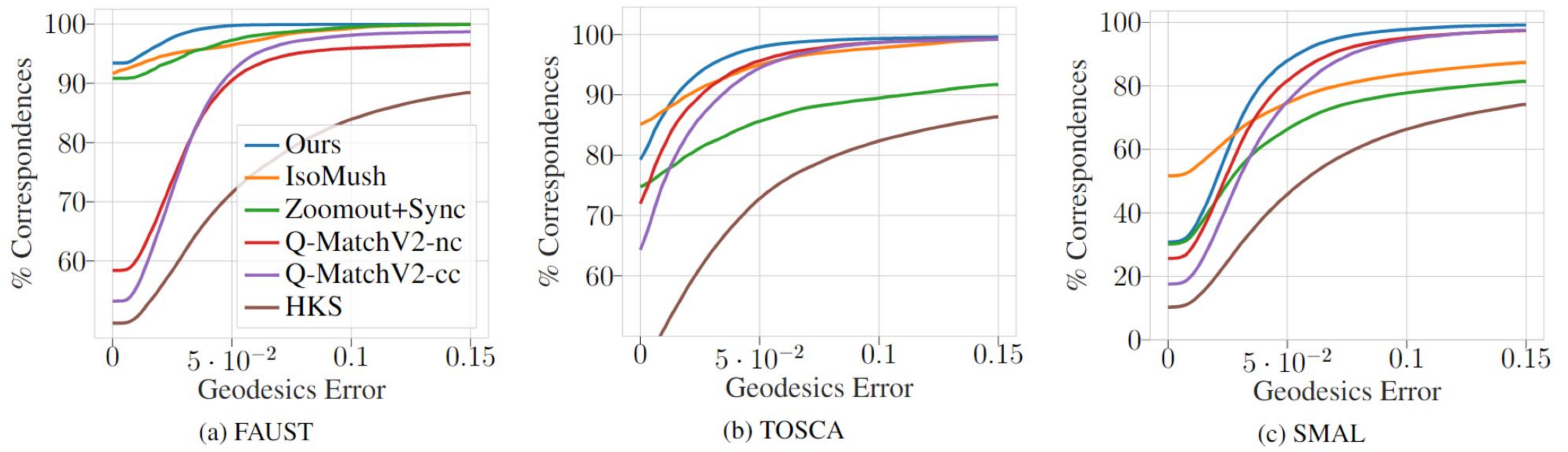


Experimental Results

Given the ground-truth correspondences $P^*_{\mathcal{IJ}}$, the geodesic error of vertex *v* is:

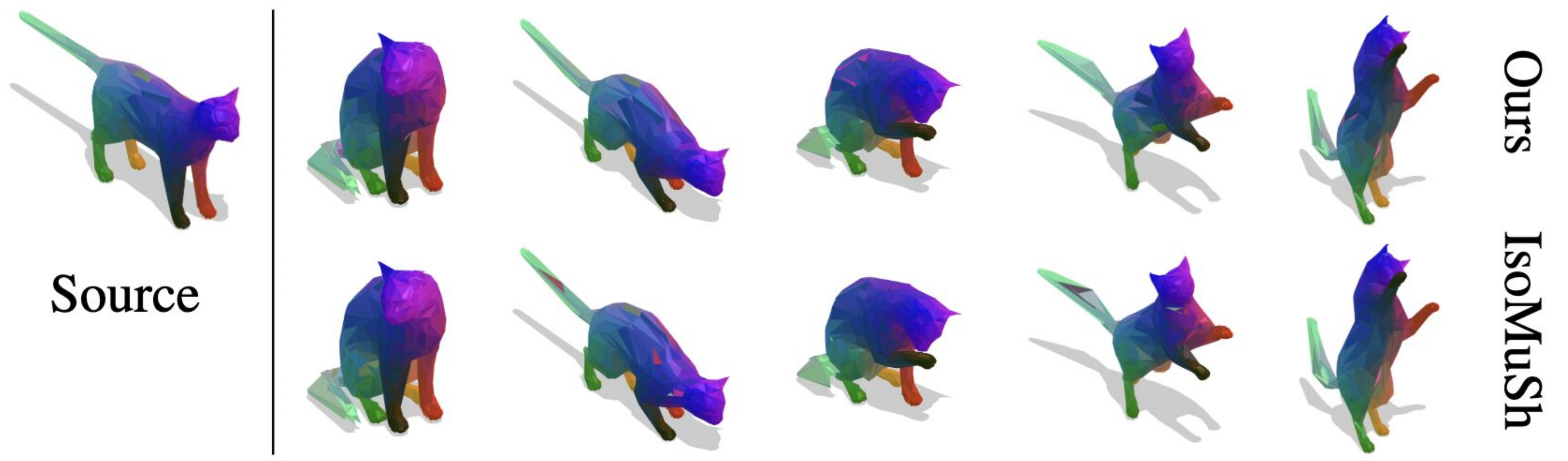
$$e_v(P_{\mathcal{IJ}}) = \frac{d^g_{\mathcal{J}}(v^\top P_{\mathcal{IJ}}, v^\top P^*_{\mathcal{IJ}})}{\mathrm{diam}(\mathcal{J})}$$

PCK curves plot the fraction of errors (y-axis) that is below a threshold (x-axis):



The area-under-the-curve (AUC) of these plots summarises them:

	Ours	QMatchV2-cc	QMatchV2-nc	IsoMuSh[3]	ZoomOut[2]	HKS
FAUST	**0.989**	0.886	0.879	*0.974*	0.886	0.746
TOSCA	**0.967**	0.932	0.940	*0.952*	0.864	0.742
SMAL	*0.866*	0.771	0.813	**0.926**	0.851	0.544



Conclusion

- Our method is comparable to classical methods, which is observed for the first time in the literature
- Approximating certain higher-order terms still allows for high-quality solutions, which is promising for future quantum methods
- Iterative optimisation of triplets is highly effective and classical multi-matching methods might benefit as well

References

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[2] Melzi et al. Spectral upsampling for efficient shape correspondence. In ACM TOG, 2019.
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Code