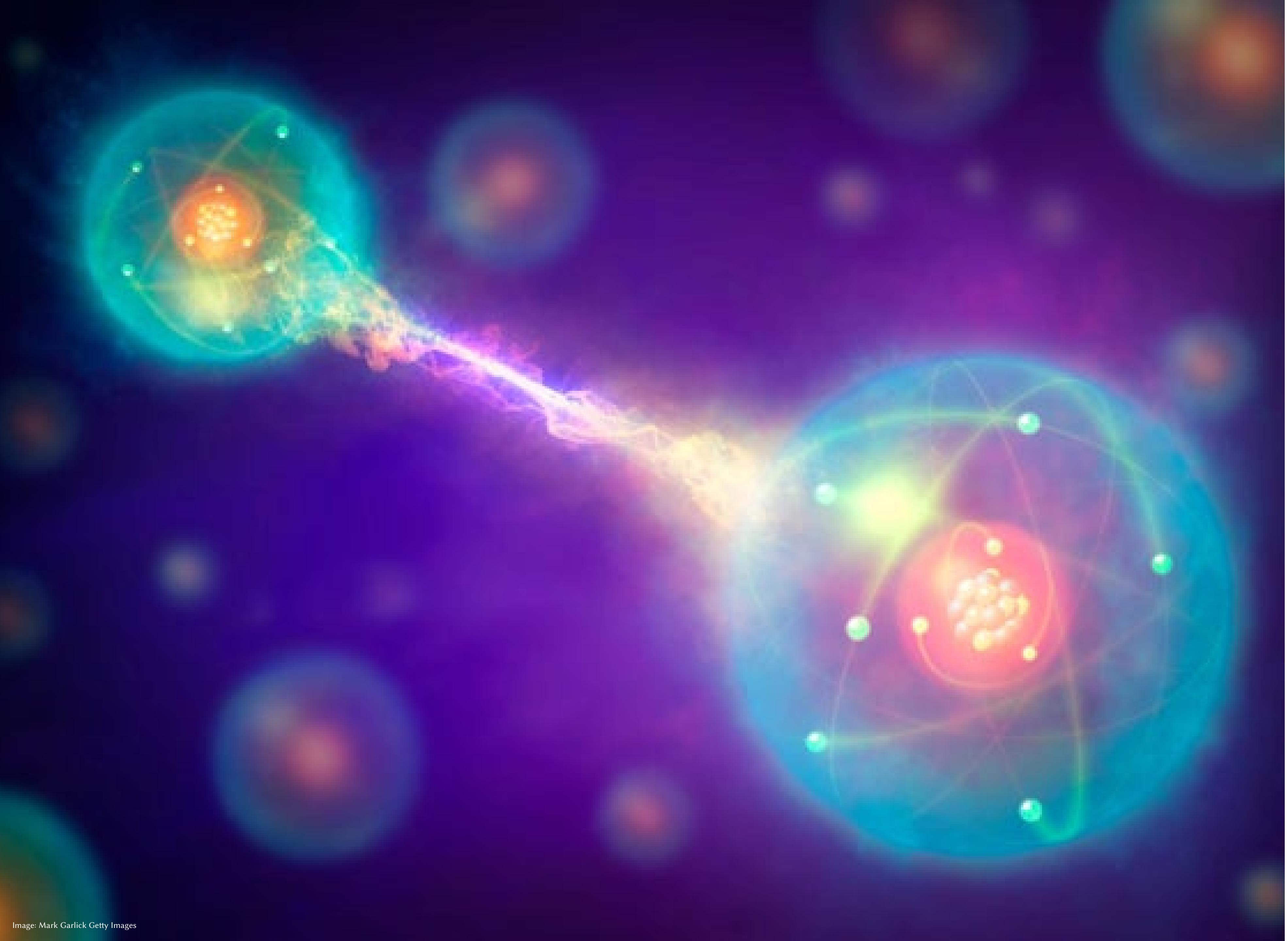

Image: Mark Garlick Getty Images

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Visual Computing and AI Department

4D and Quantum Vision Group

Motus • Quanta • Visus

SIC Lecture Series

01.09.2021

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3D Computer Vision:
From a Classical to a Quantum Perspective

SIC Lecture Series (01.09.2021)

Vladislav Golyanik

4D and Quantum Vision Group

Motus • Quanta • Visus

Visual Computing and
AI Department

Outline

- Introduction
- Overview of the Research Fields (4DQV Group)
- Adiabatic Quantum Computing
- Quantum Algorithms for Computer Vision and Graphics

Why 3D+t?

Images: NASA/WMAP Science Team/ Art by Dana Berry; NASA/ADLER/U. CHICAGO/WESLEYAN/JPL-CALTECH/TSP/SCHMIDT; Event Horizon Telescope (EHT)

Why 3D+t?

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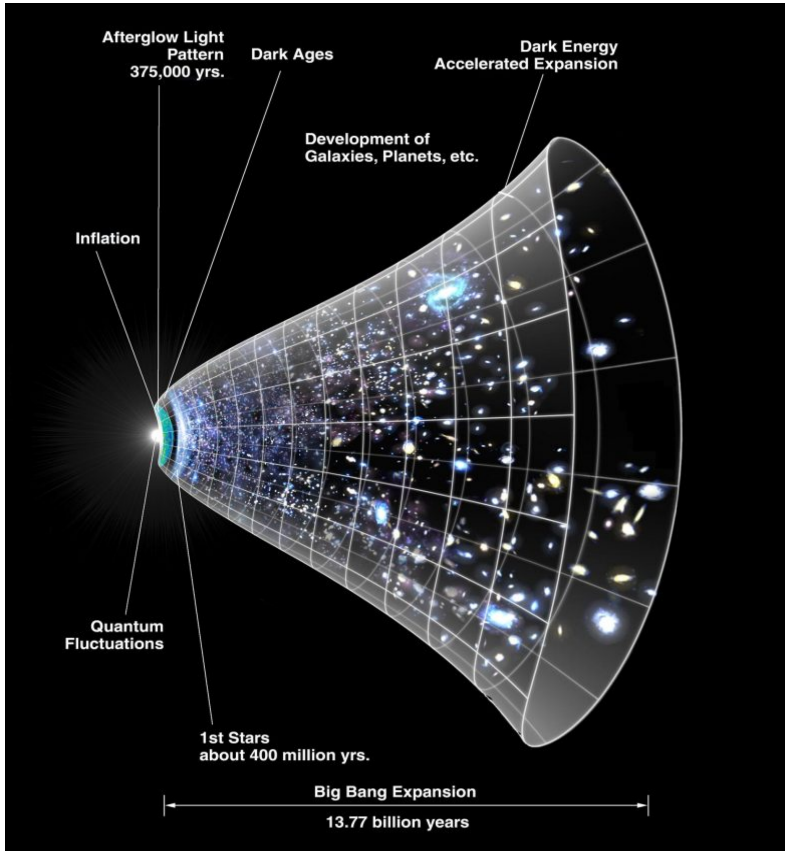

Expansion of the universe

Why 3D+t?

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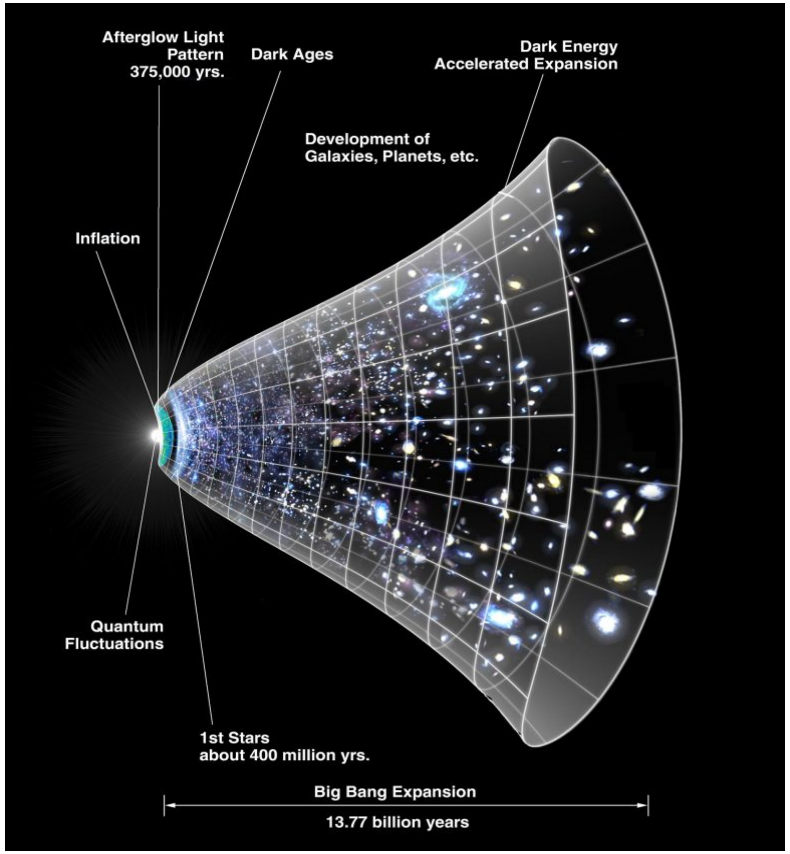

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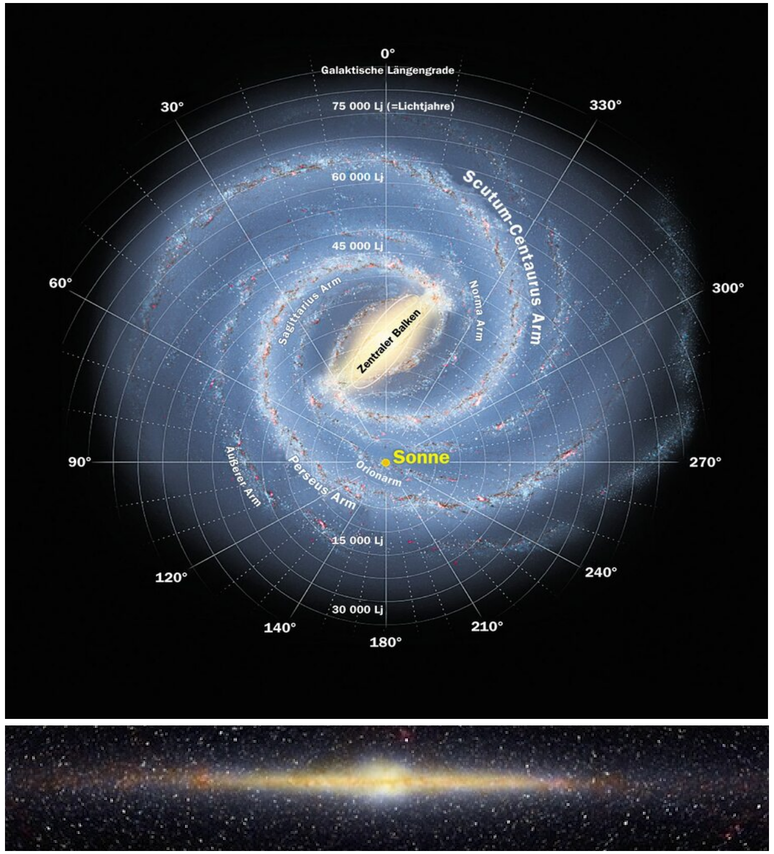

The Milky Way

Images: NASA/WMAP Science Team/ Art by Dana Berry; NASA/ADLER/U. CHICAGO/WESLEYAN/JPL-CALTECH/TSP/SCHMIDT; Event Horizon Telescope (EHT)

Why 3D+t?

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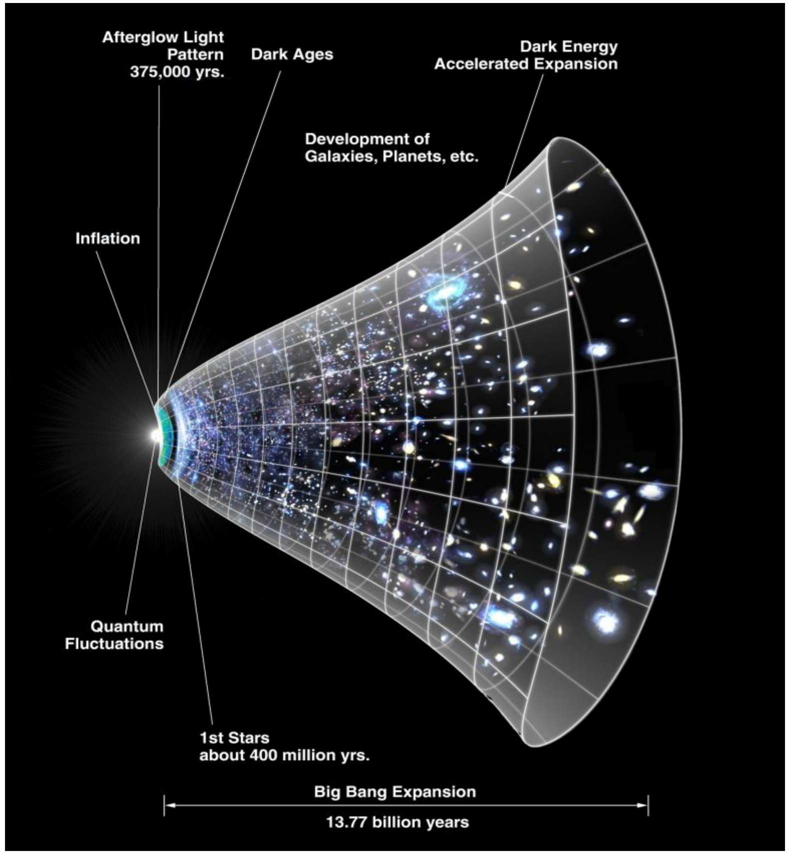

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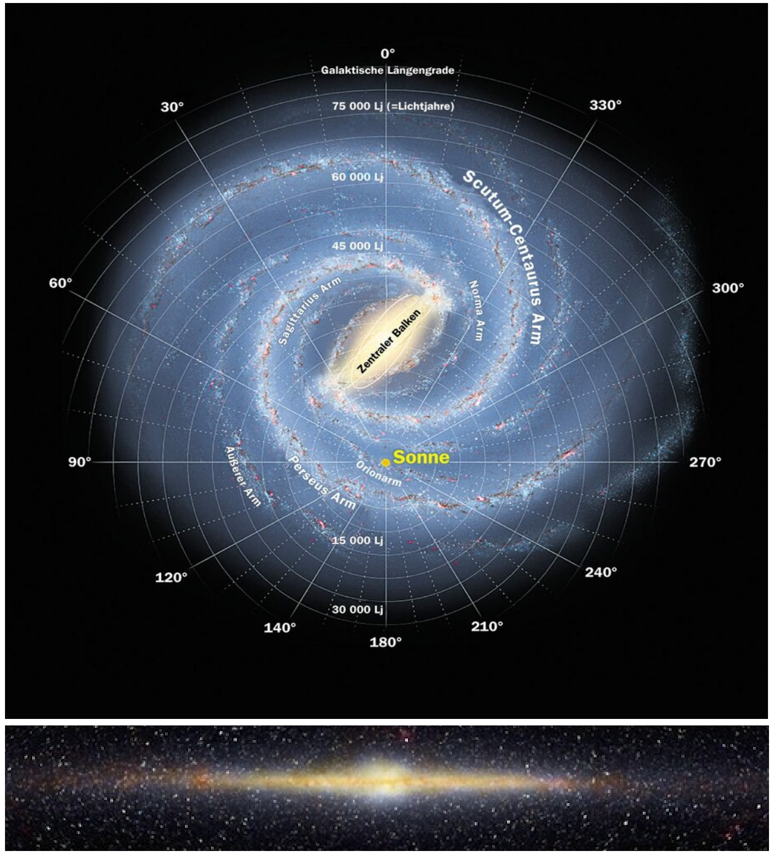


The Milky Way

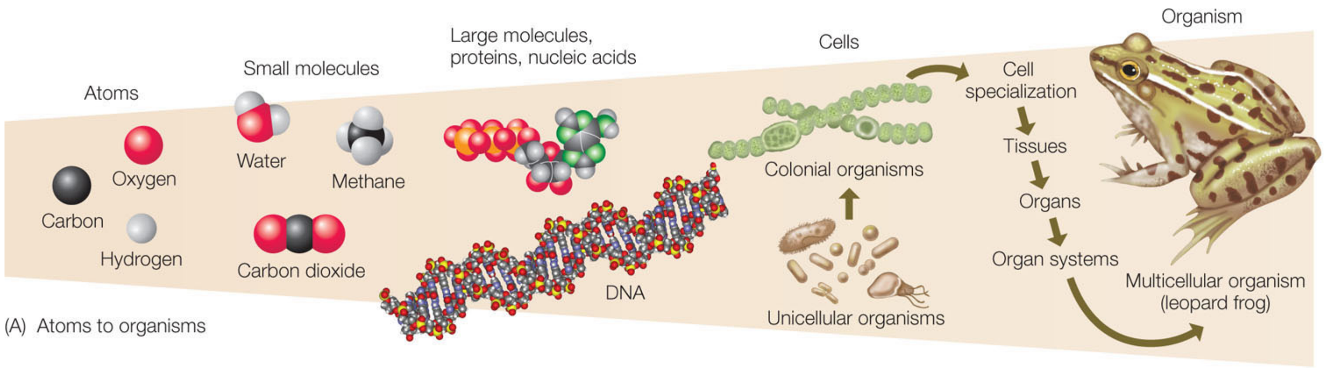
Black Hole M87*

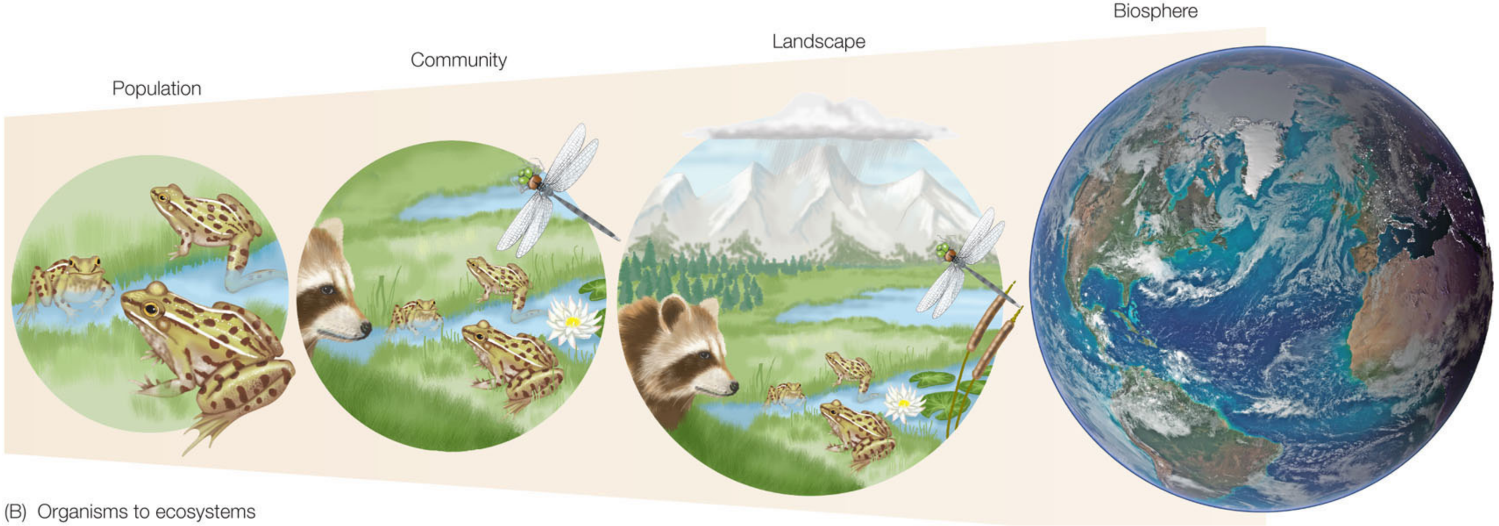
The history of the universe is, in effect, a huge and ongoing quantum computation. The universe is a quantum computer.

Seth Lloyd

Images: NASA/WMAP Science Team/ Art by Dana Berry; NASA/ADLER/U. CHICAGO/WESLEYAN/JPL-CALTECH/TSP/SCHMIDT; Event Horizon Telescope (EHT)

Why 3D+t?





1.5 *Biosphere*: NASA images by Reto Stöckli, based on data from NASA and NOAA.

Tewari *et al.* High-Fidelity Monocular Face Reconstruction based on an Unsupervised Model-based Face Autoencoder. TPAMI, 2017.
Parot *et al.* Photometric Stereo Endoscopy. Journal of Biomedical Optics, 2013.
Golyanik *et al.* Introduction to Coherent Depth Fields for Dense Monocular Surface Recovery. BMVC, 2017.

Why 3D+t?

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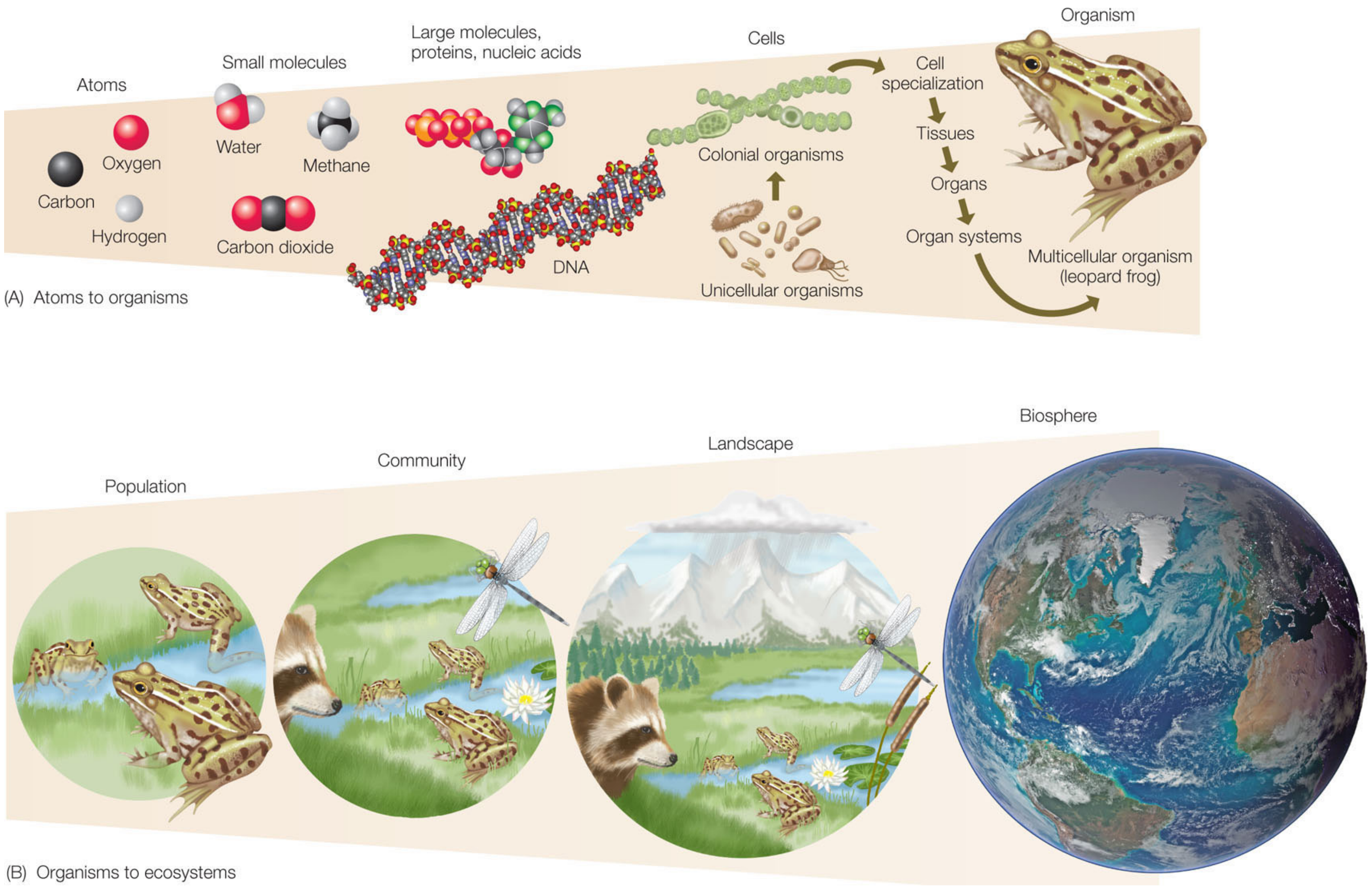
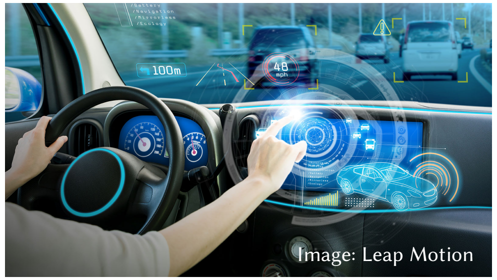


1.5 *Biosphere*: NASA images by Reto Stöckli, based on data from NASA and NOAA.

Sports



HCI (Gesture recognition)

Tewari *et al.* High-Fidelity Monocular Face Reconstruction based on an Unsupervised Model-based Face Autoencoder. TPAMI, 2017.
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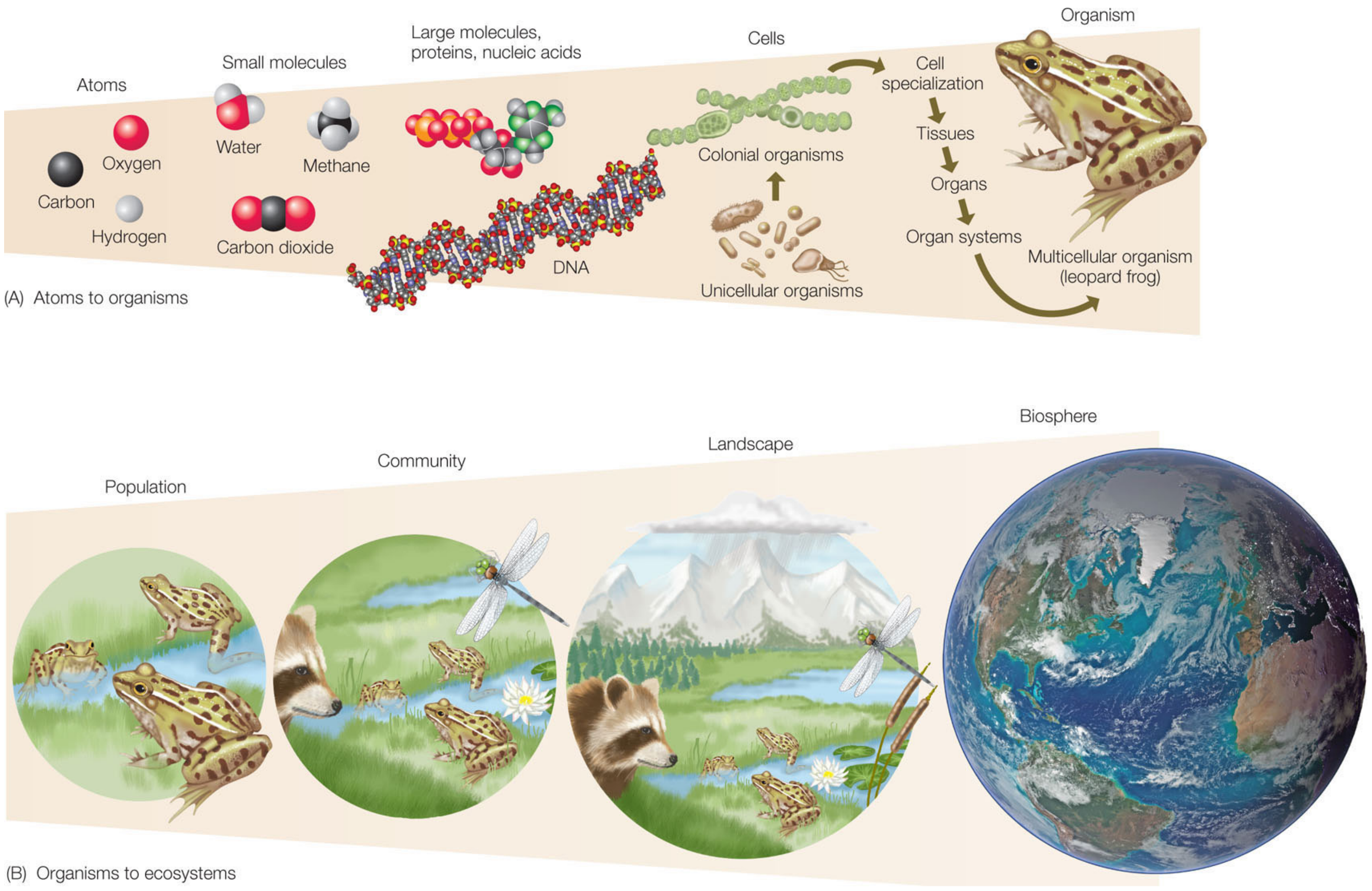

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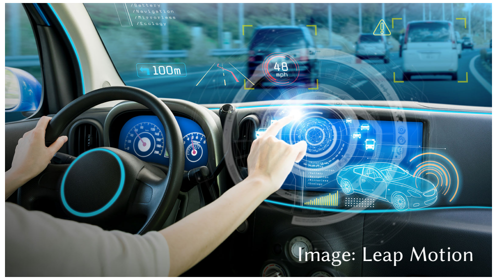

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Human face and body parts (*e.g.*, back)

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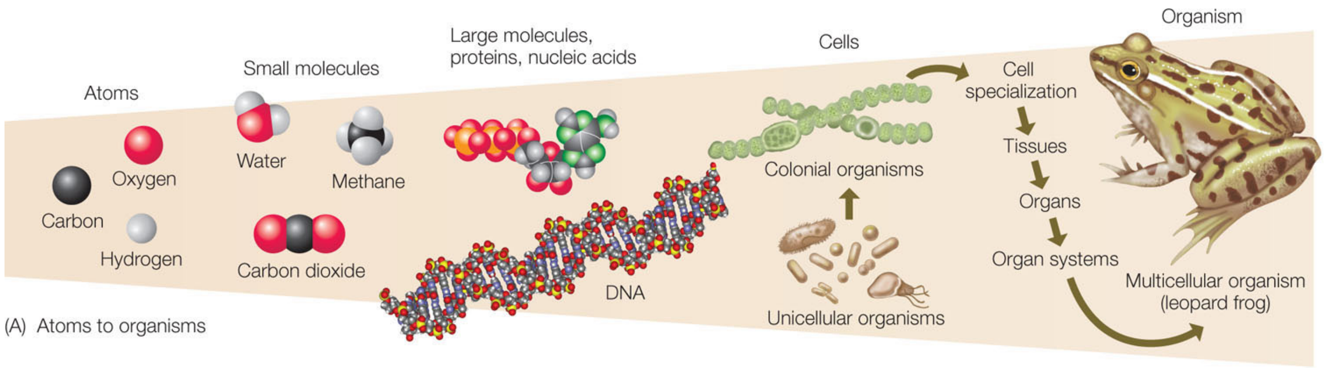
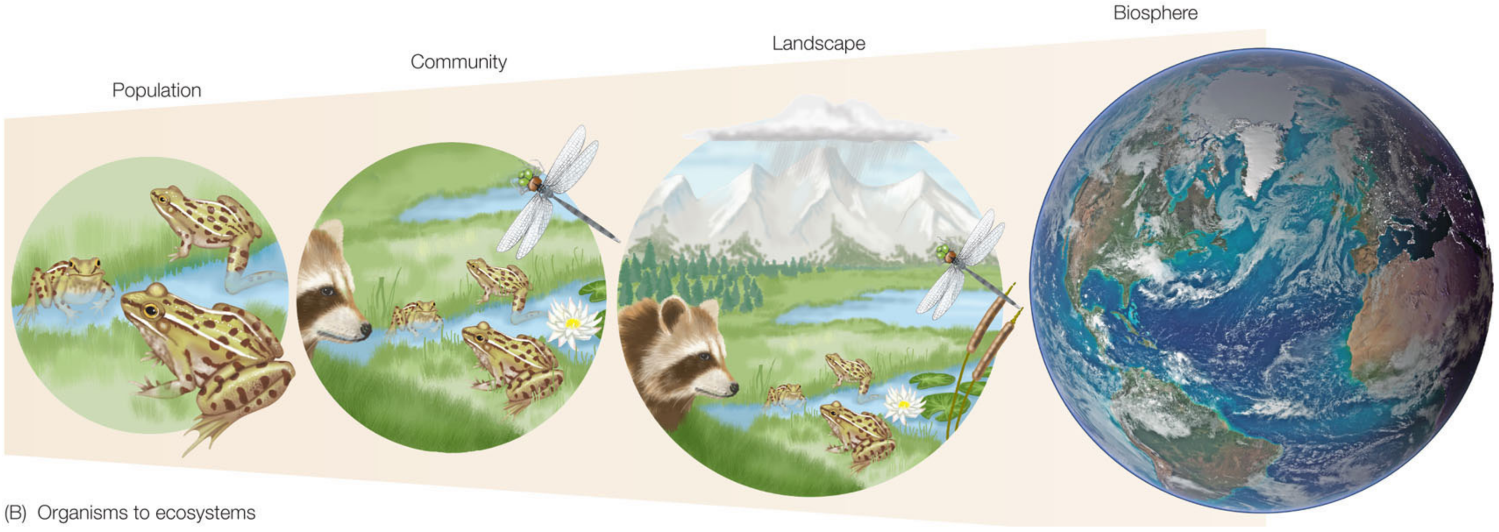


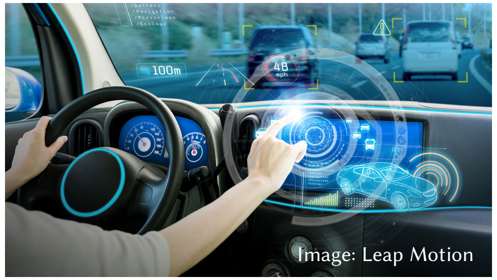
(A) Atoms to organisms



(B) Organisms to ecosystems

1.5 *Biosphere*: NASA images by Reto Stöckli, based on data from NASA and NOAA.

Sports



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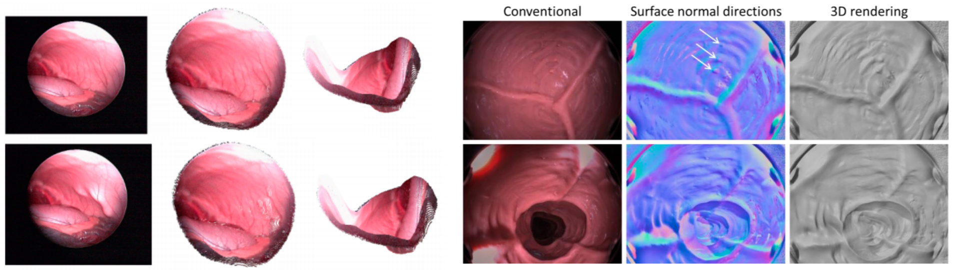

Endoscopic videos

Tewari *et al.* High-Fidelity Monocular Face Reconstruction based on an Unsupervised Model-based Face Autoencoder. TPAMI, 2017.
Parot *et al.* Photometric Stereo Endoscopy. Journal of Biomedical Optics, 2013.
Golyanik *et al.* Introduction to Coherent Depth Fields for Dense Monocular Surface Recovery. BMVC, 2017.

Applications of Quantum Computing

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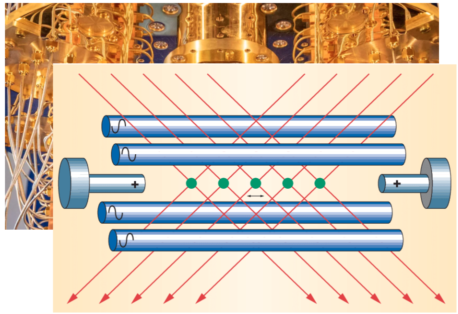
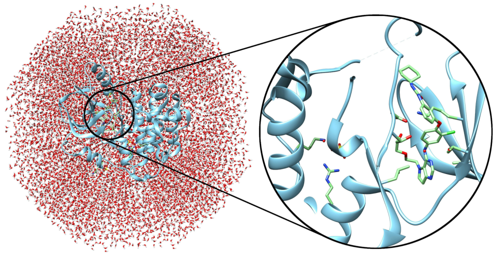
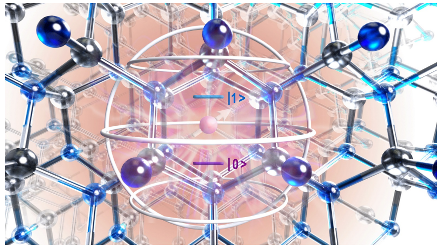
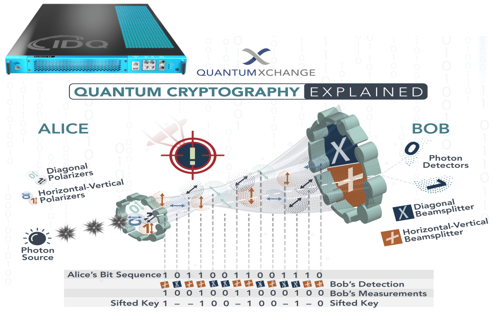




Material science



Drug discovery



Quantum cryptography

TensorFlow Quantum

Cirq

Machine learning

intel IBM D-Wave The Quantum Computing Company 1QBit QCWARE ISARA SECURITY MEASURES FOR THE QUANTUM AGE STRANGE WORKS IONQ ColdQuanta The Quantum Atomics Company Q-CTRL rigetti Google AI Quantum

Boosters:

- Theoretically obtained results are promising
- Broad interest of scientific community and industry
- Progress in physical hardware realisations
- Investments

Applications:

- Material science
- Drug research
- Cryptography
- Machine learning

…

+ **many others**

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https://www.profacgen.com/Quantum-Mechanics-Chemistry-in-Drug-Design.htm
https://www.anl.gov/article/solving-materials-problems-with-a-quantum-computer
https://quantumxc.com/quantum-cryptography-explained/
https://www.idquantique.com/quantum-safe-security/products/cerberis-xg-qkd-system/
A close-up view of the IBM Q quantum computer. The processor is in the silver-colored cylinder. Stephen Shankland/CNET
Steane and Rieffel. Beyond Bits: The Future of Quantum Information Processing. Computer, 2000.

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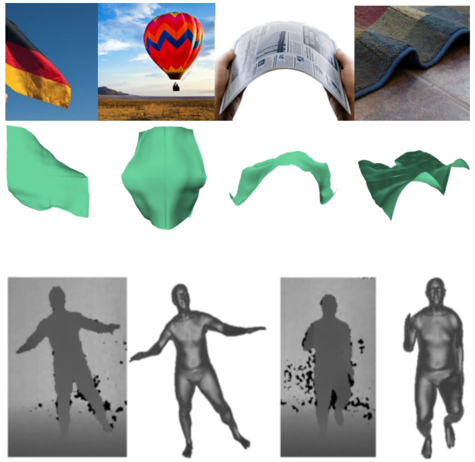
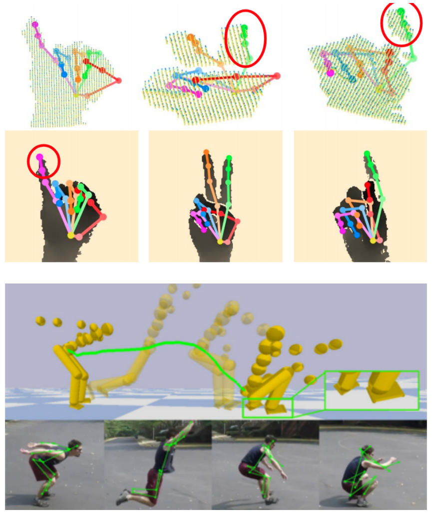
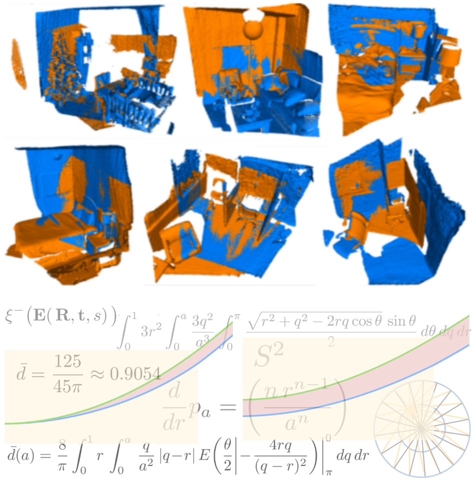
Overview: 4DQV Group (D6)



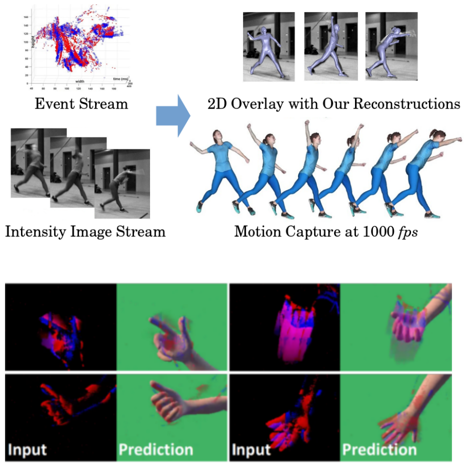
4D reconstruction (general non-rigid scenes)



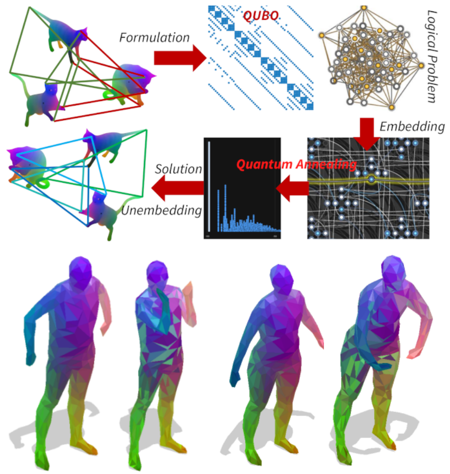
4D reconstruction (hand and humans)



Point cloud analysis (alignment)



Event-based vision (non-rigid scenes)

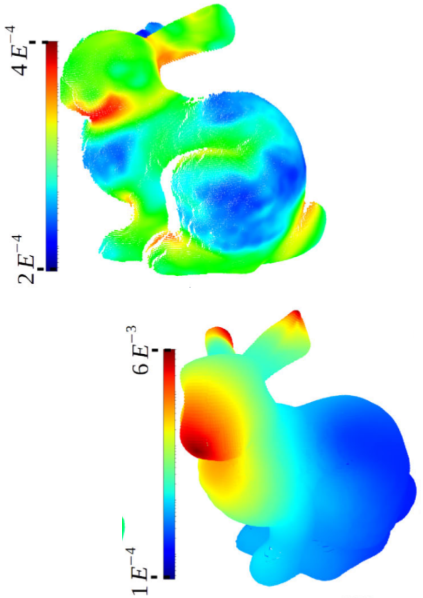


Quantum CV

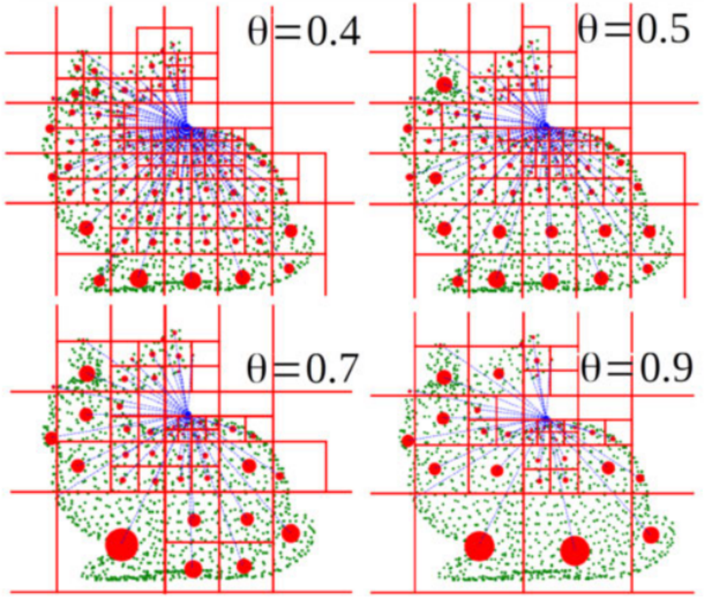
- 4D reconstruction and tracking of non-rigid scenes and objects
- Neural rendering
- Point set registration and other matching problems
- Event-based approaches in vision and graphics
- Quantum algorithms for computer vision and graphics

Neural rendering (implicit 3D)

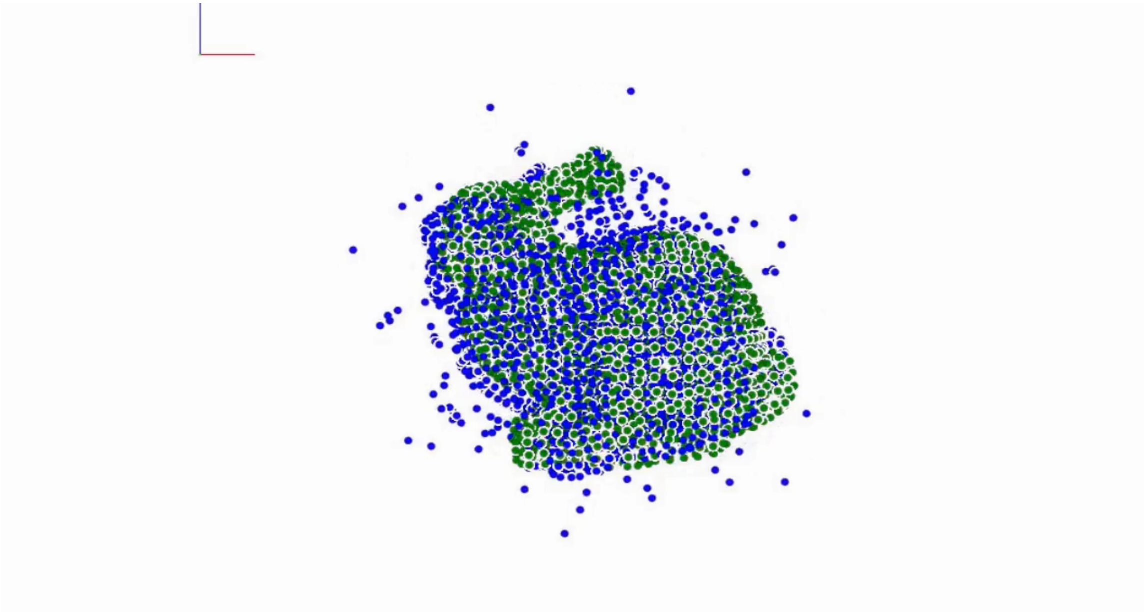
Fast Gravitational Approach (FGA)



Pointwise features
mapped to masses



Barnes-Hut 2^D-tree

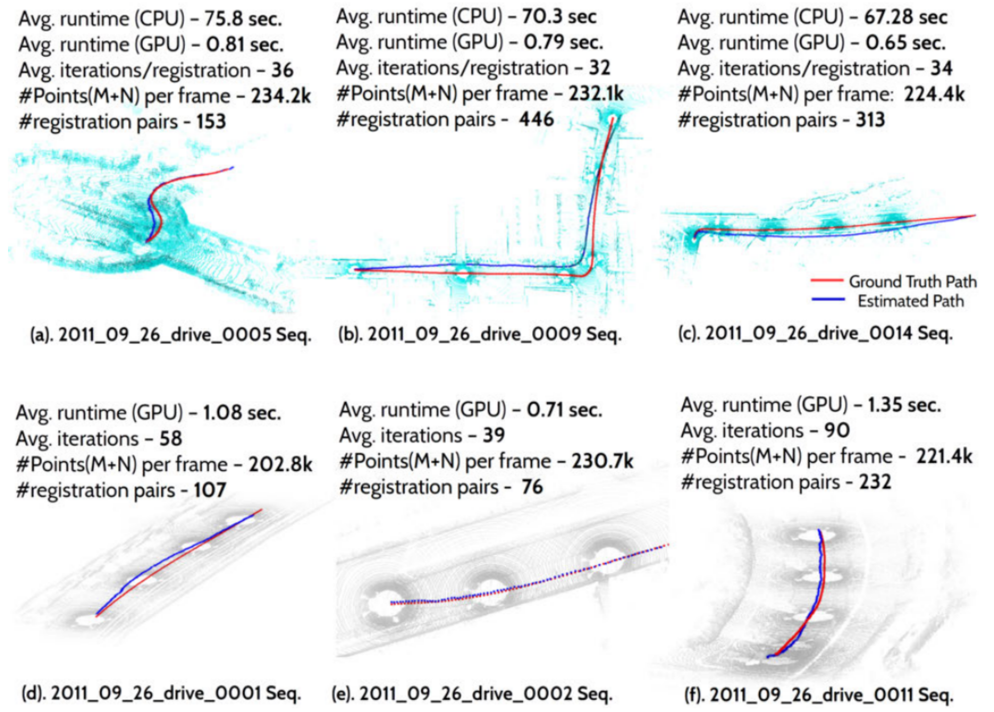


Alignment iterations (FGA)

Gravitational Potential Energy (GPE) of the system: $\mathbf{E}(\mathbf{R}, \mathbf{t}) = -G \sum_{i,j} \frac{m_{\mathbf{Y}_i} m_{\mathbf{X}_j}}{(\|\mathbf{R}\mathbf{r}_{\mathbf{Y}_i} + \mathbf{t} - \mathbf{r}_{\mathbf{X}_j}\| + \epsilon)}$

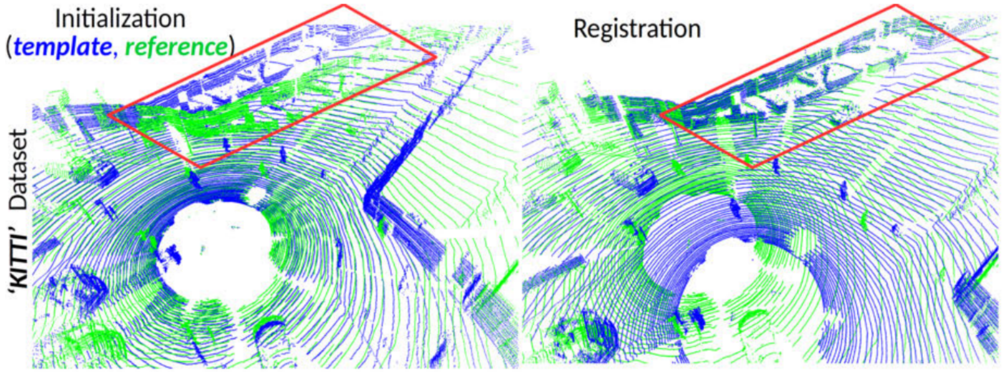
Ali, Kahraman, Stricker, Theobalt, Golyanik. Fast Gravitational Approach for Rigid Point Set Registration With Ordinary Differential Equations. *IEEE Access*, 2021.

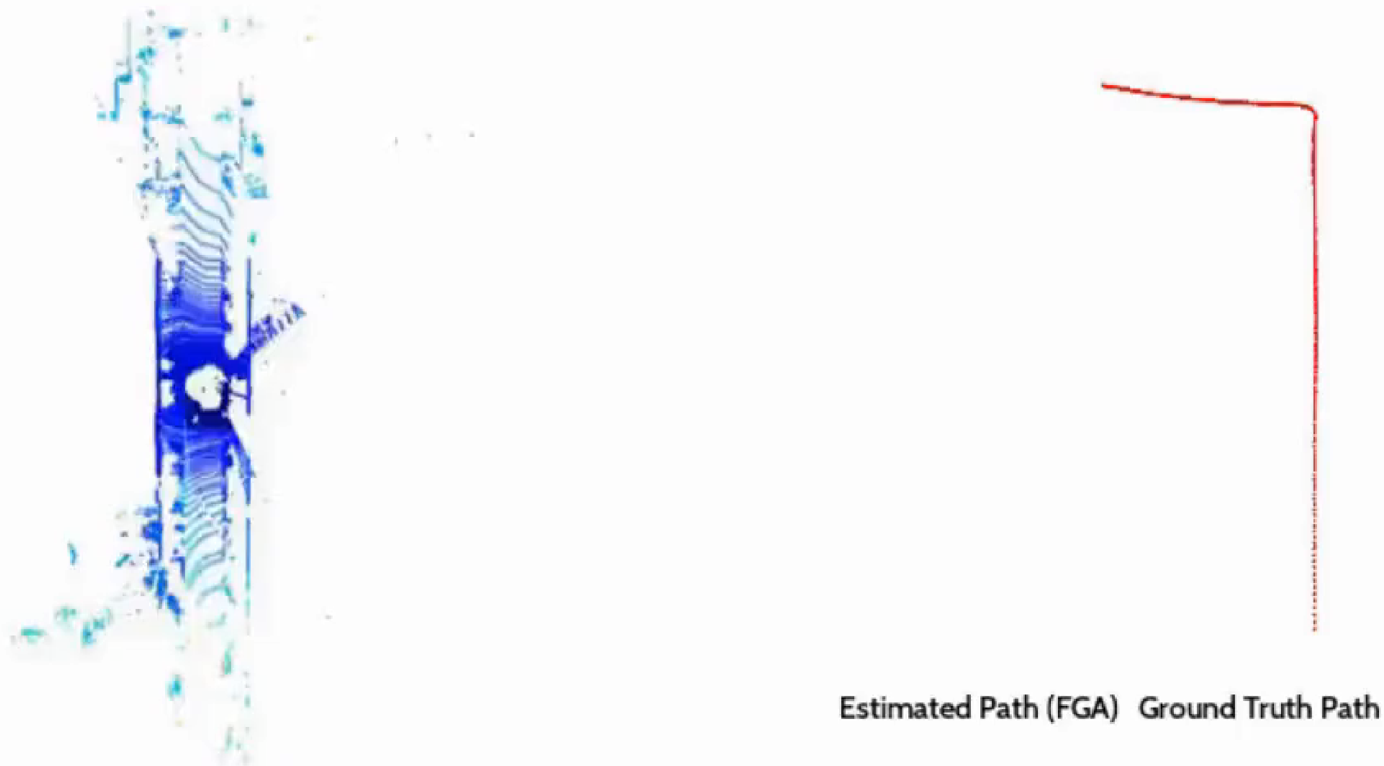
Alignment of LIDAR Data



(a). 2011_09_26_drive_0005 Seq. (b). 2011_09_26_drive_0009 Seq. (c). 2011_09_26_drive_0014 Seq.

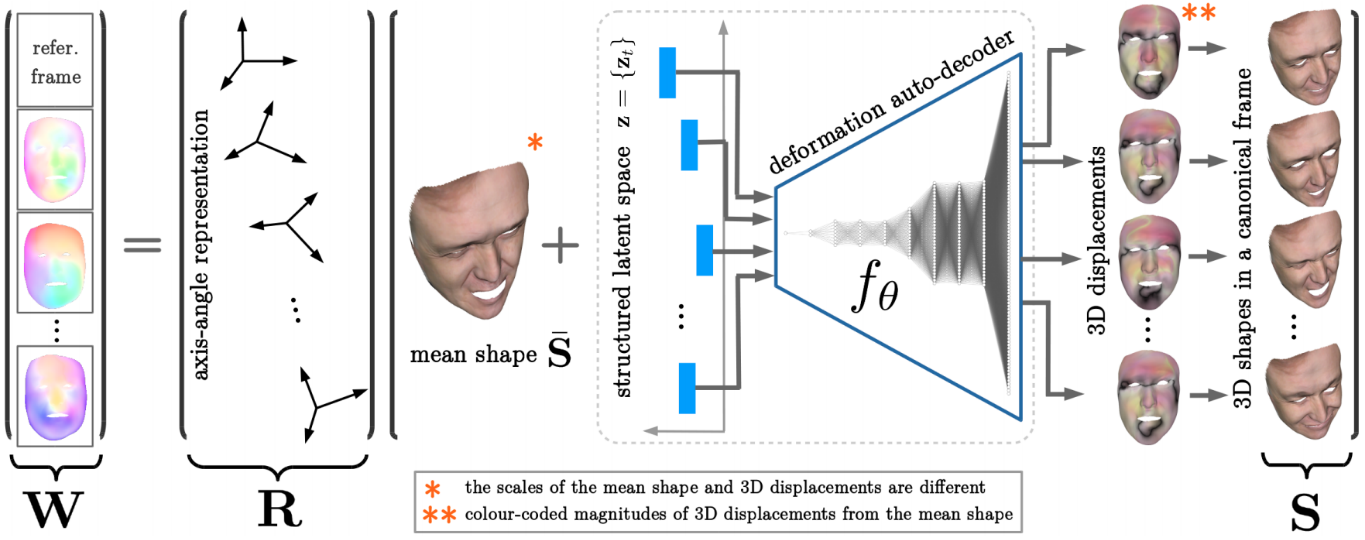
(d). 2011_09_26_drive_0001 Seq. (e). 2011_09_26_drive_0002 Seq. (f). 2011_09_26_drive_0011 Seq.





Path Estimation using the sensor poses (T^*) obtained while registering consecutive frames

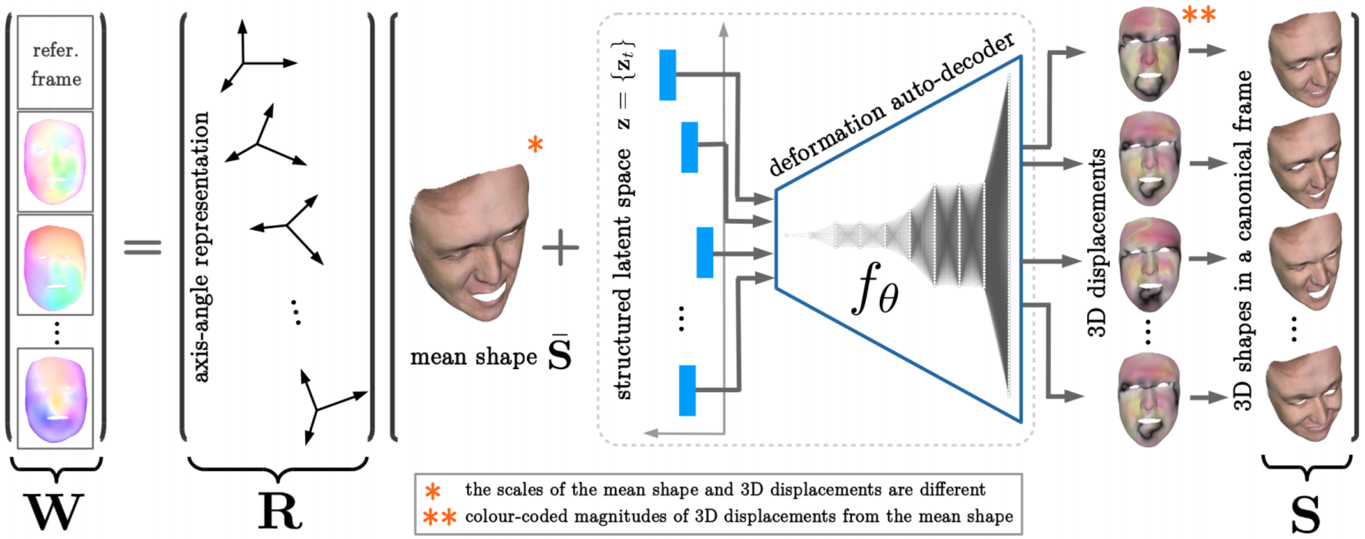
Neural Dense NRSfM



Overview of neural dense NRSfM with latent space constraints

Sidhu, Tretschk, Golyanik, Agudo and Theobalt. Neural Dense Non-Rigid Structure from Motion with Latent Space Constraints. *ECCV*, 2020.

Neural Dense NRSfM



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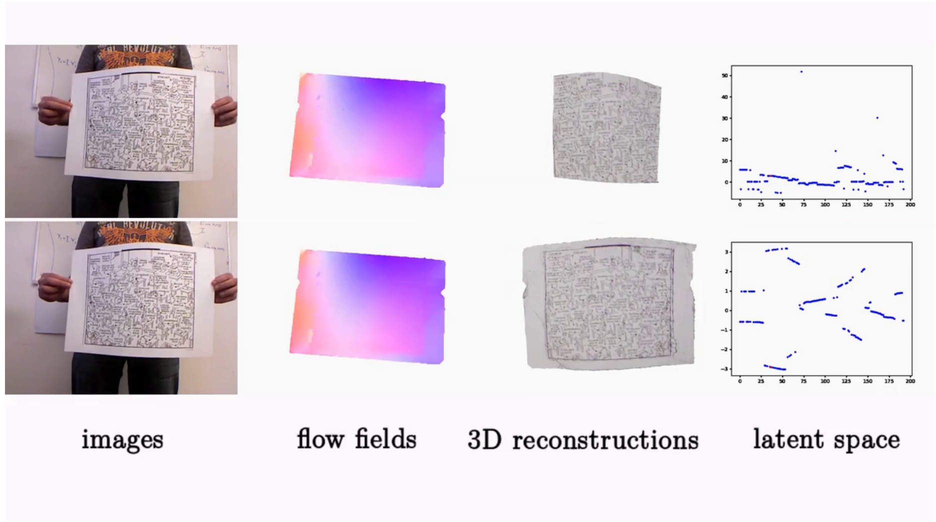


3D reconstructions of *Kinect t-shirt* and *paper* sequences

optical flow colour scheme

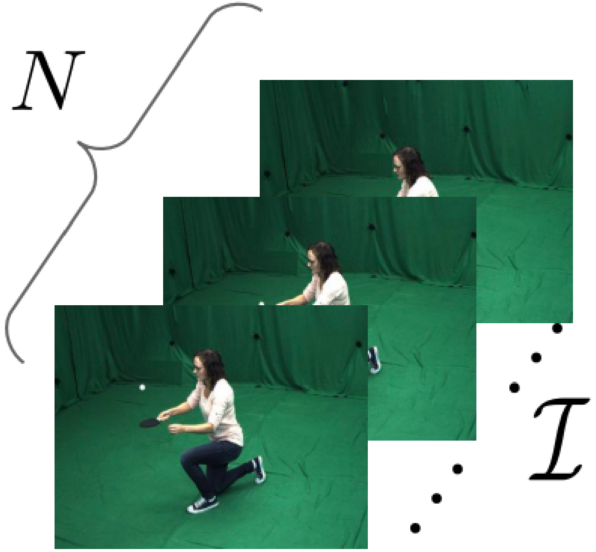
Sparsity constraints on the latent space function:

$$\mathbf{E}_{\text{latent}}(\mathbf{z}) = \|\mathcal{F}(\mathbf{z})\|_1$$

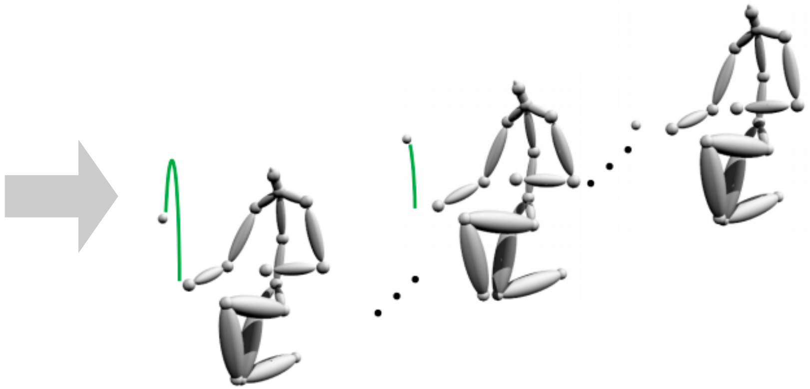
- Dense NRSfM is worth studying because the principles are applicable to many other (more well-posed) problems
- Results of dense NRSfM serve as a “lower bound” for many other problems

Sidhu, Tretschk, Golyanik, Agudo and Theobalt. Neural Dense Non-Rigid Structure from Motion with Latent Space Constraints. *ECCV*, 2020.

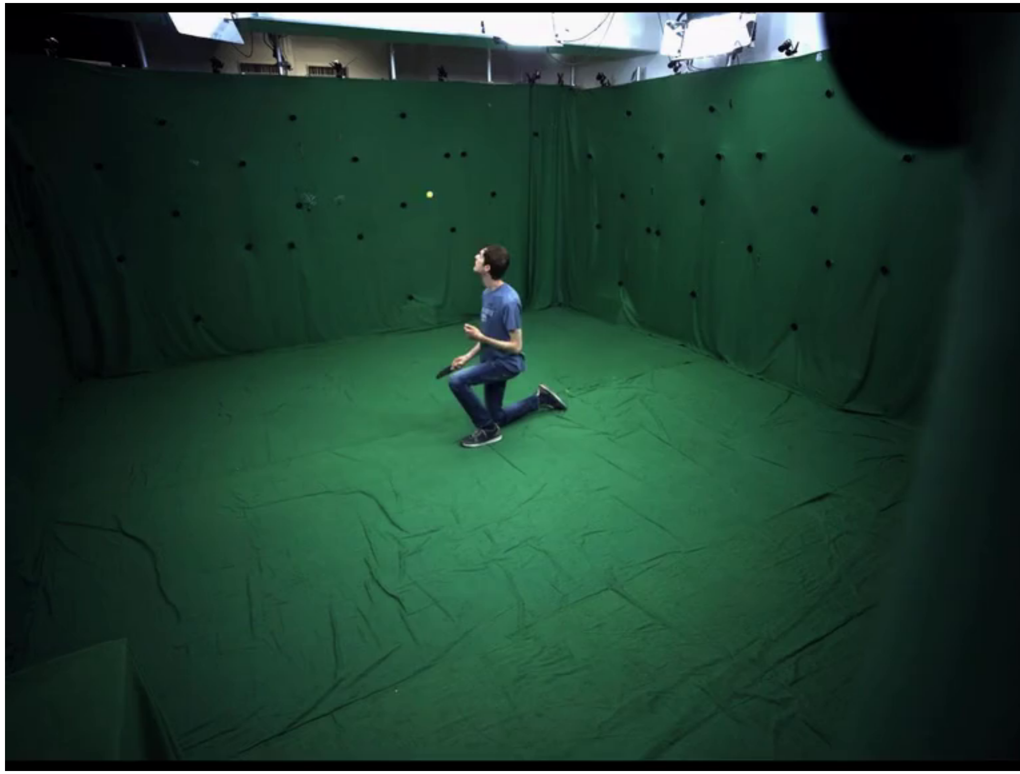
GraviCap



Input 2D images



3D reconstructions
(human and object)



Input videos and the corresponding 3D reconstructions

Dabral, Shimada, Jain, Theobalt, Golyanik. Gravity-Aware Monocular 3D Human-Object Reconstruction. *ICCV*, 2021.

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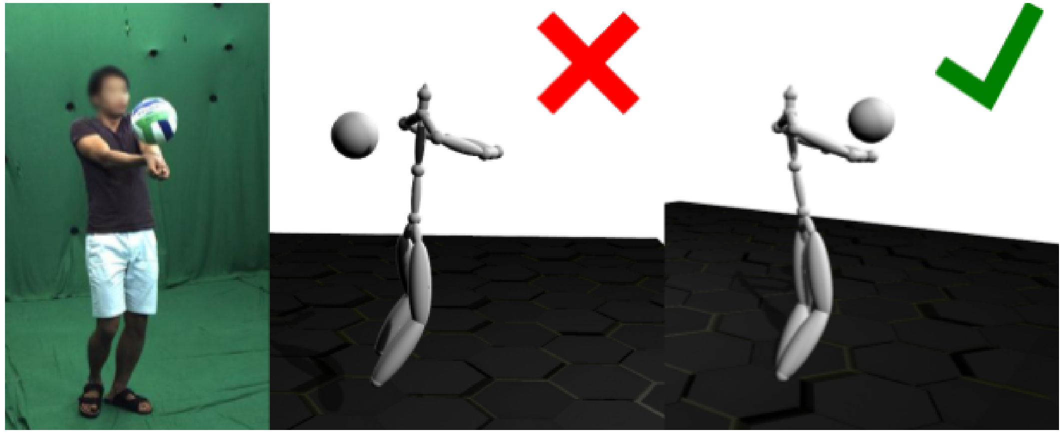
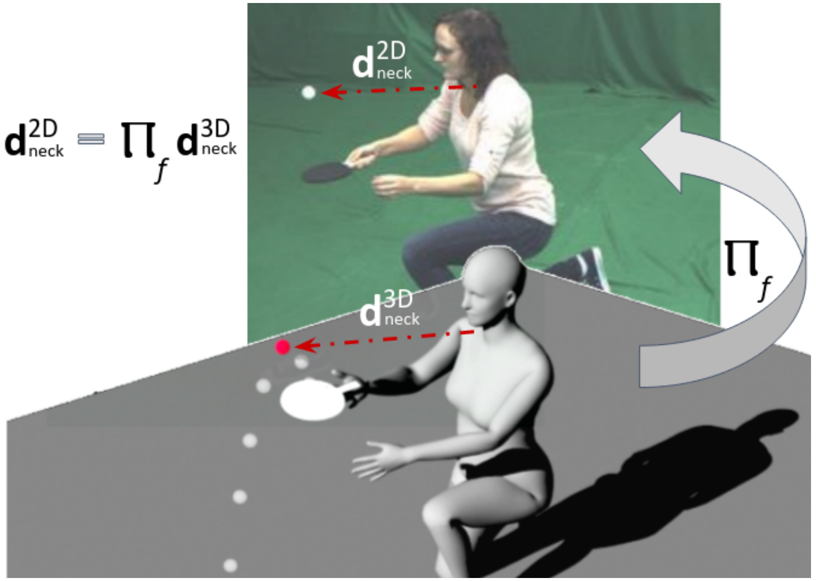
GraviCap





$$x_i = f\frac{X_i}{Z_i} + c_x,\ y_i = f\frac{Y_i}{Z_i} + c_y,\ \forall i,$$

$$\text{s. t.}\quad \sqrt{g_x^2 + g_y^2 + g_z^2} = 9.81\, m/s^2,$$

$$\text{where}\ \begin{cases} X_i = X_0 + u_x t + \frac{1}{2} g_x t^2, \\ Y_i = Y_0 + u_y t + \frac{1}{2} g_y t^2,\ \text{and} \\ Z_i = Z_0 + u_z t + \frac{1}{2} g_z t^2. \end{cases}$$

Dabral, Shimada, Jain, Theobalt, Golyanik. Gravity-Aware Monocular 3D Human-Object Reconstruction. *ICCV*, 2021.

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EventHands

inivation

DAVIS 240C

2021 ICCV OCTOBER 11-17 VIRTUAL

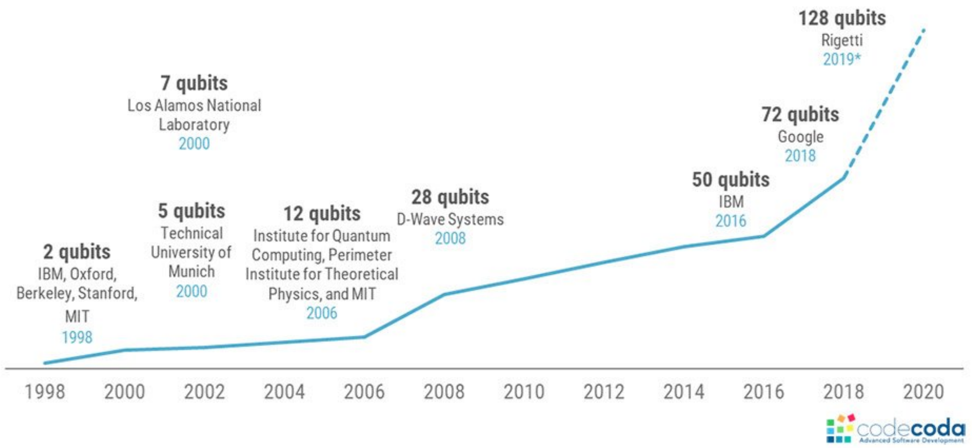
Playback speed: 0.5x

Rudnev, Golyanik, Wang, Mueller, Seidel, Elgharib, Theobalt. EventHands: Real-Time Neural 3D Hand Pose Estimation from an Event Stream. *ICCV*, 2021.

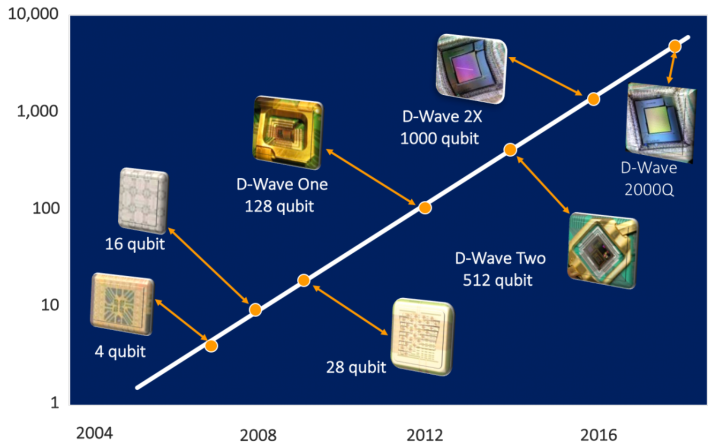
Part II: Changing the Perspective

- Introduction
- Overview of the Research Fields (4DQV Group)
- Adiabatic Quantum Computing
- Quantum Algorithms for Computer Vision and Graphics

Circuit-Based *vs* Adiabatic Model

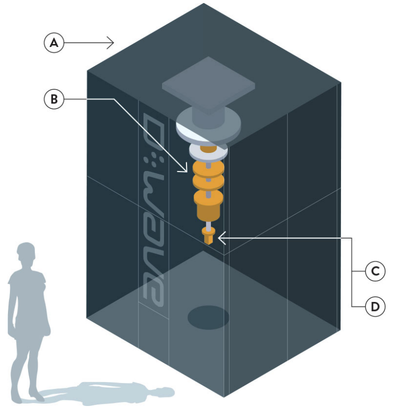


qubits (all QC types)



qubits and AQC generations (D-Wave)

	Circuit-based QC	Adiabatic QC (Quantum Annealers)
type	Universal	Specialised
can solve	All classical and quantum algorithms (for the circuit-based model)	QUBO/Ising problems
number of qubits	up to 27	>5000 (35000 couplers)

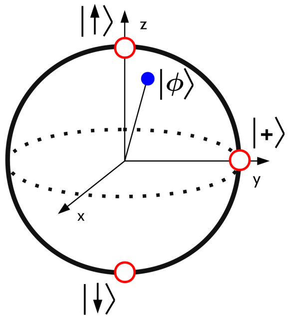


Images: https://codecoda.com/en/blog/entry/understanding-quantum-computing; D-Wave Systems

Operations on Qubits

complex numbers

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$



Greenberger, Horne, Zeilinger. Going Beyong Bell's Theorem. Fundamental Theories of Physics, 1989.
https://qiskit.org/documentation/tutorials/circuits/01_circuit_basics.html
https://www.leifiphysik.de/atomphysik/quantenmech-atommodell/versuche/schroedingers-katze-ein-gedankenexperiment

Operations on Qubits

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complex numbers

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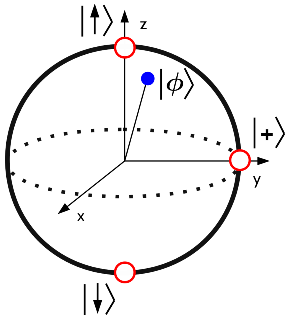
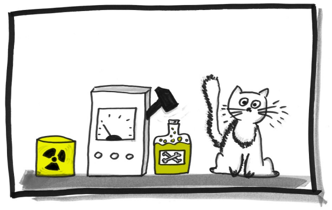


Born's rule:

$|\alpha|^2$ to obtain $|0\rangle$

$|\beta|^2$ to obtain $|1\rangle$

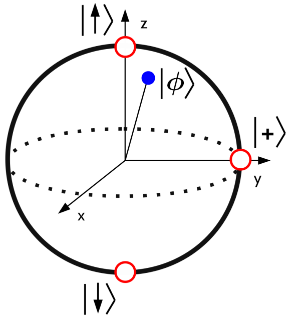


Greenberger, Horne, Zeilinger. Going Beyong Bell's Theorem. Fundamental Theories of Physics, 1989.
https://qiskit.org/documentation/tutorials/circuits/01_circuit_basics.html
https://www.leifiphysik.de/atomphysik/quantenmech-atommodell/versuche/schroedingers-katze-ein-gedankenexperiment

Operations on Qubits

complex numbers

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$



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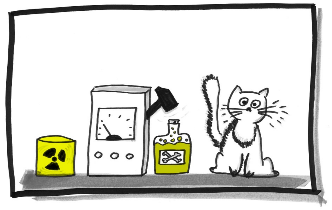

Bra–Ket (Dirac) notation:

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

$$\langle 0| = \begin{bmatrix}1 & 0\end{bmatrix} \qquad \langle 1| = \begin{bmatrix}0 & 1\end{bmatrix}$$

$$\langle q_1|q_2\rangle = \begin{bmatrix}\alpha & \beta\end{bmatrix}\begin{bmatrix}\eta\\ \zeta\end{bmatrix} = \alpha\eta + \beta\zeta$$

Greenberger, Horne, Zeilinger. Going Beyong Bell's Theorem. Fundamental Theories of Physics, 1989.
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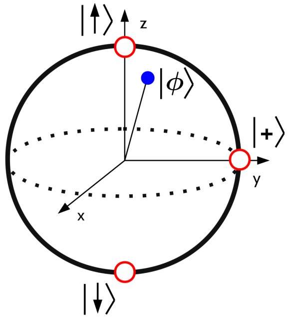
https://www.leifiphysik.de/atomphysik/quantenmech-atommodell/versuche/schroedingers-katze-ein-gedankenexperiment

Operations on Qubits

complex numbers

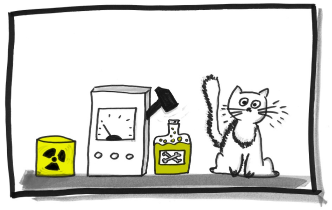
$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$



Born's rule:

$|\alpha|^2$ to obtain $|0\rangle$

$|\beta|^2$ to obtain $|1\rangle$



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$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

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Hilbert spaces (2^n-dimensional vector spaces):

$$|0\rangle \otimes |1\rangle = |01\rangle = \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \; |00\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \; |10\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \; |11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

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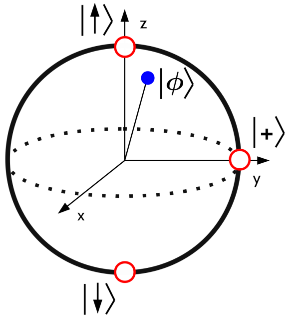

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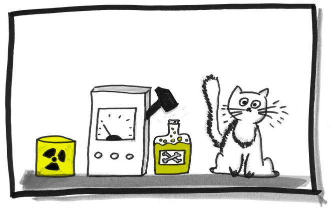

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QC is a form of reversible computing:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix} \qquad CNOT = CX = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$$

* acts on a single qubit

* acts on two qubits

- No loss of information
- Unitary matrices preserving vector lengths

Greenberger, Horne, Zeilinger. Going Beyong Bell's Theorem. Fundamental Theories of Physics, 1989.
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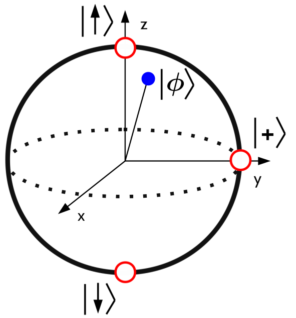

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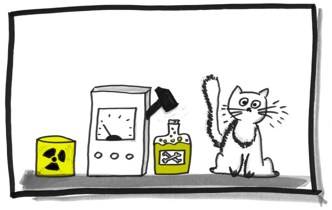

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* acts on a single qubit

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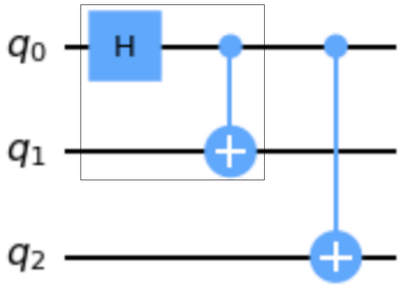
* acts on two qubits

- No loss of information
- Unitary matrices preserving vector lengths

Qubit entanglement:



$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Bell state

$$|\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

A three-qubit GHZ state

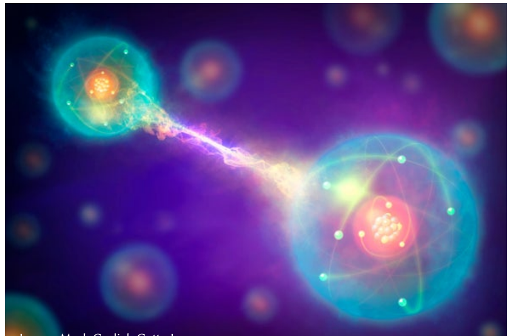


Image: Mark Garlick Getty Images

- Entangled particles *communicate* with each other instantaneously
- No *external* information can passed when entangled particles affect each other

Greenberger, Horne, Zeilinger. Going Beyong Bell's Theorem. Fundamental Theories of Physics, 1989.
https://qiskit.org/documentation/tutorials/circuits/01_circuit_basics.html
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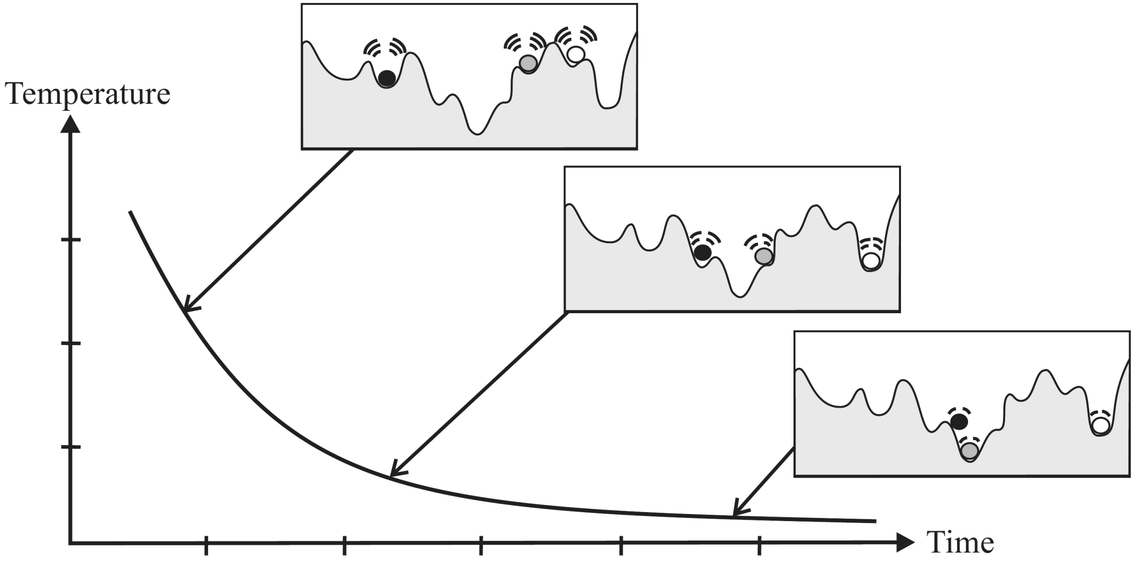
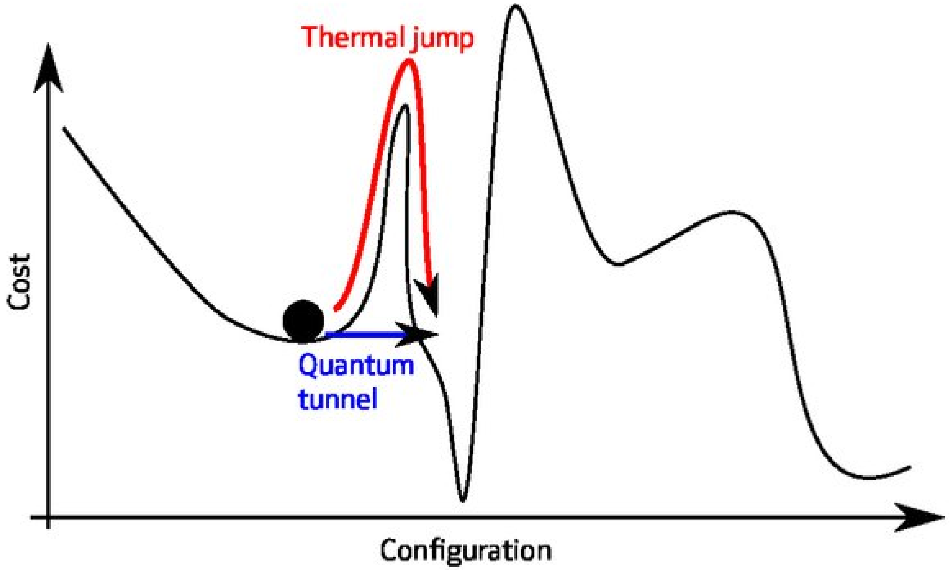
Simulated *vs* Quantum Annealing





Simulated Thermal Fluctuations

Main Parameter: Temperature

Quantum Fluctuations (physical phenomenon)

Main Parameter: Transverse Magnetic Field

Effects: Tunnelling, Superposition and Entanglement

Kirkpatrick et al. Optimization by Simulated Annealing. Science, 1983.
Ledesma et al. Simulated Annealing Evolution. Open access peer-reviewed chapter. https://www.intechopen.com/chapters/38520
Steiger et al. Performance of quantum annealing hardware. Electro-Optical and Infrared Systems: Technology and Applications XII; and Quantum Information Science and Technology, 2015.

Transition between Hamiltonians

Initial state: $$|\psi(t=0)\rangle = \bigotimes_{i=1}^{n} \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle\right)$$

Transition (simplified): $$H(t) = \left(1 - \frac{t}{\tau}\right) H_I + \frac{t}{\tau} H_P$$

Final state encoding the problem and the data: $$\boxed{\min_{s \in \{-1,1\}^n} \; s^\top J s + b^\top s}$$

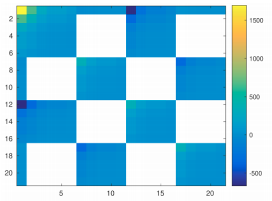
binary vector
(qubits during optimisation)

qubit couplings
(interaction weights)

qubit biases
(individual weights)

	q1	q2	q3
q1		0.7	1.9
q2	0.7		-0.3
q3	1.9	-0.3	

	q1	q2	q3
	0.5	0.5	0.1



Exemplary J (QUBO, 21 qubits)

Images: Willsch et al. GPU-accelerated simulations of quantum annealing and the quantum approximate optimization algorithm. ArXiv, 2021.
What is Quantum Annealing? https://docs.dwavesys.com/docs/latest/c_gs_2.html

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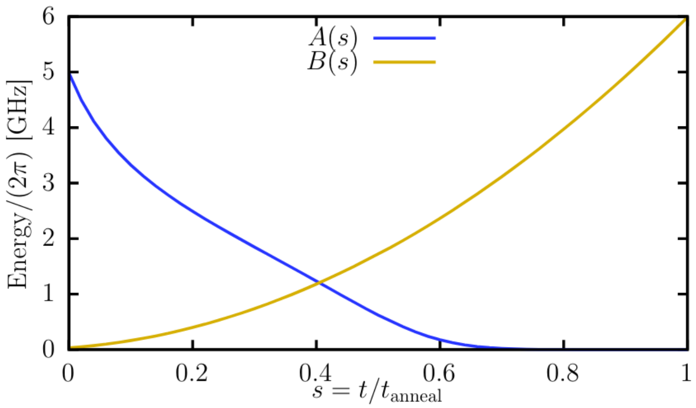

Annealing functions (schedules)

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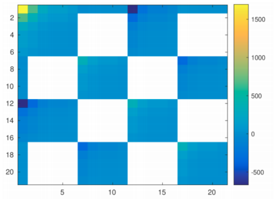
q1
q2
q3



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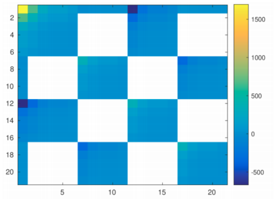

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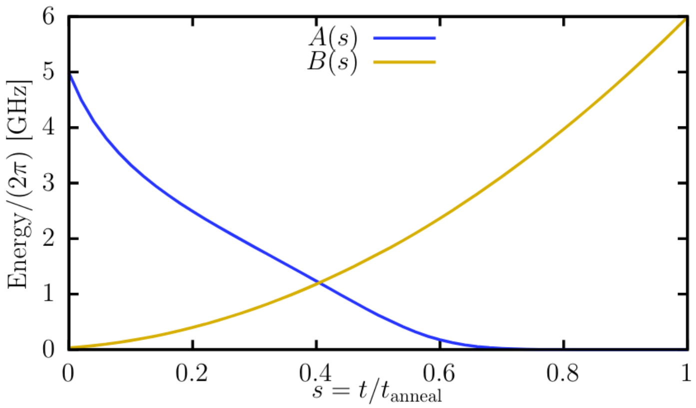

Annealing functions (schedules)

[Born and Fock, 1928]: *A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum.*

[Manin, 1980][Feynman, 1982]: A quantum computer is proposed
[Kadowaki and Nishimori, 1998]: Quantum annealing
[Farhi *et al.*, 2001]: Quantum adiabatic evolution algorithm

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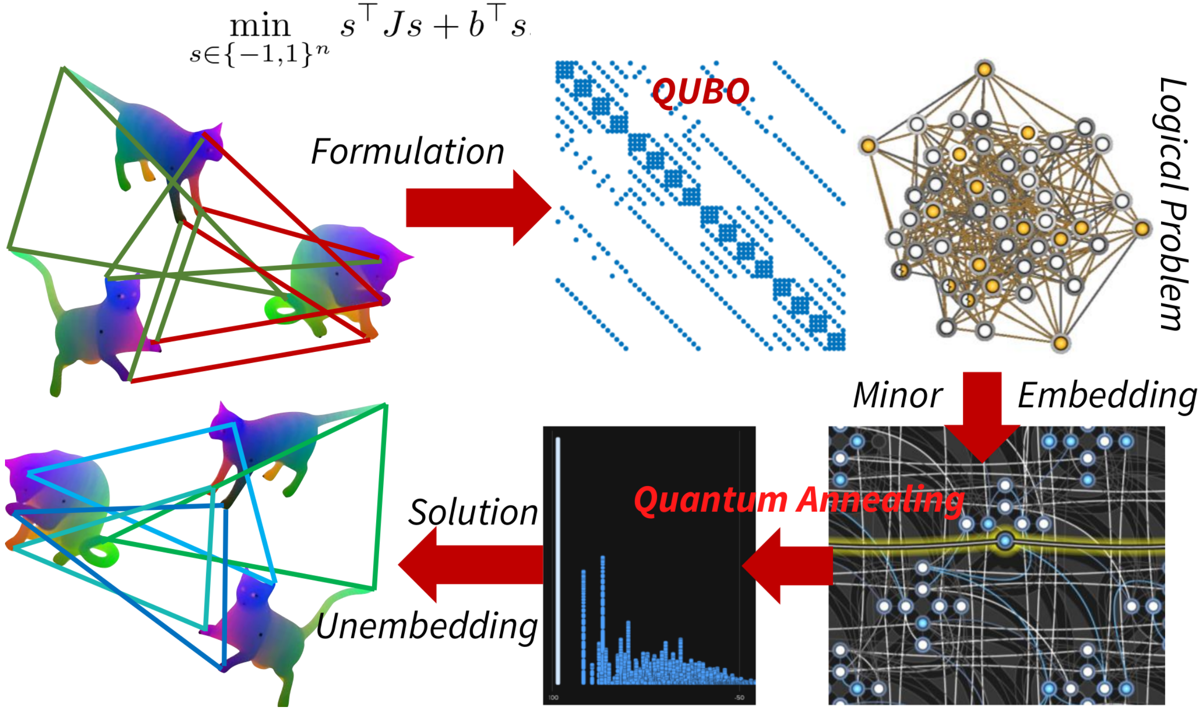
Adiabatic Quantum Algorithms



Every AQC algorithm includes six steps:

1) **QUBO preparation**
2) Minor embedding
3) **Quantum annealing (sampling)**
4) Unembedding
5) Bitstring selection
6) Solution interpretation

Birdal,* Golyanik*, Theobalt, Guibas. *Quantum Permutation Synchronization.* CVPR, 2021. * equal contribution
S. Zbinden et al. Embedding Algorithms for Quantum Annealers with Chimera and Pegasus Connection Topologies. ISC High Performance 2020.

Adiabatic Quantum Algorithms

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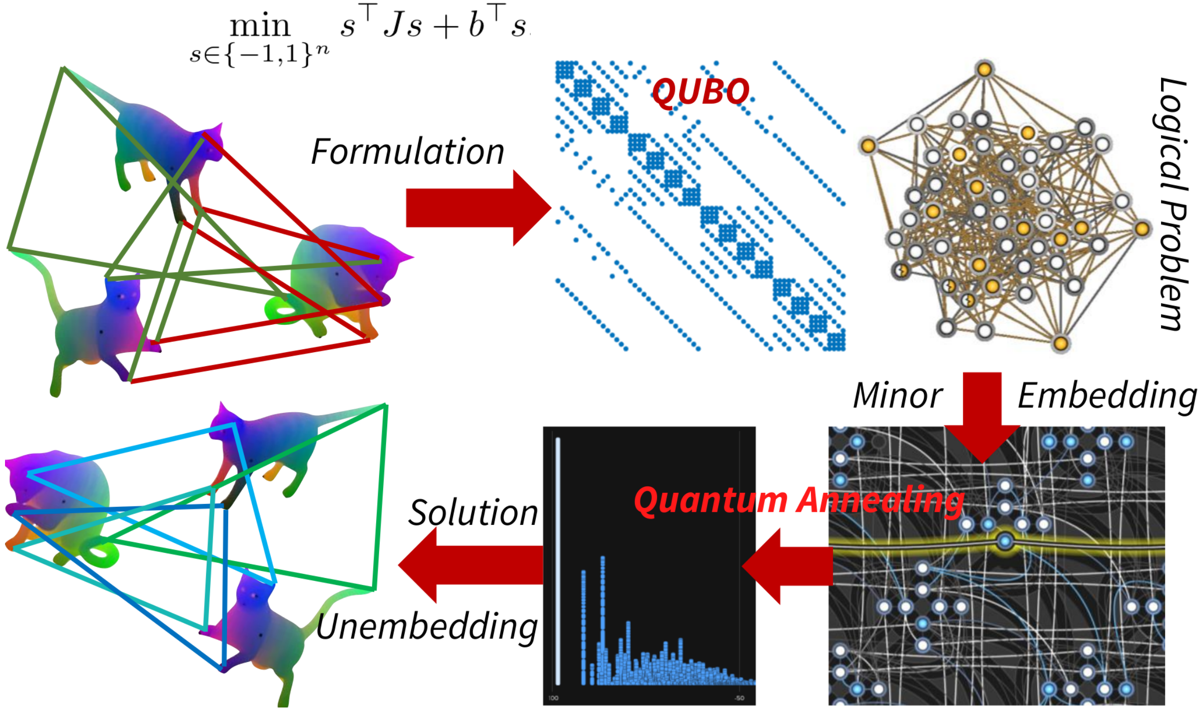

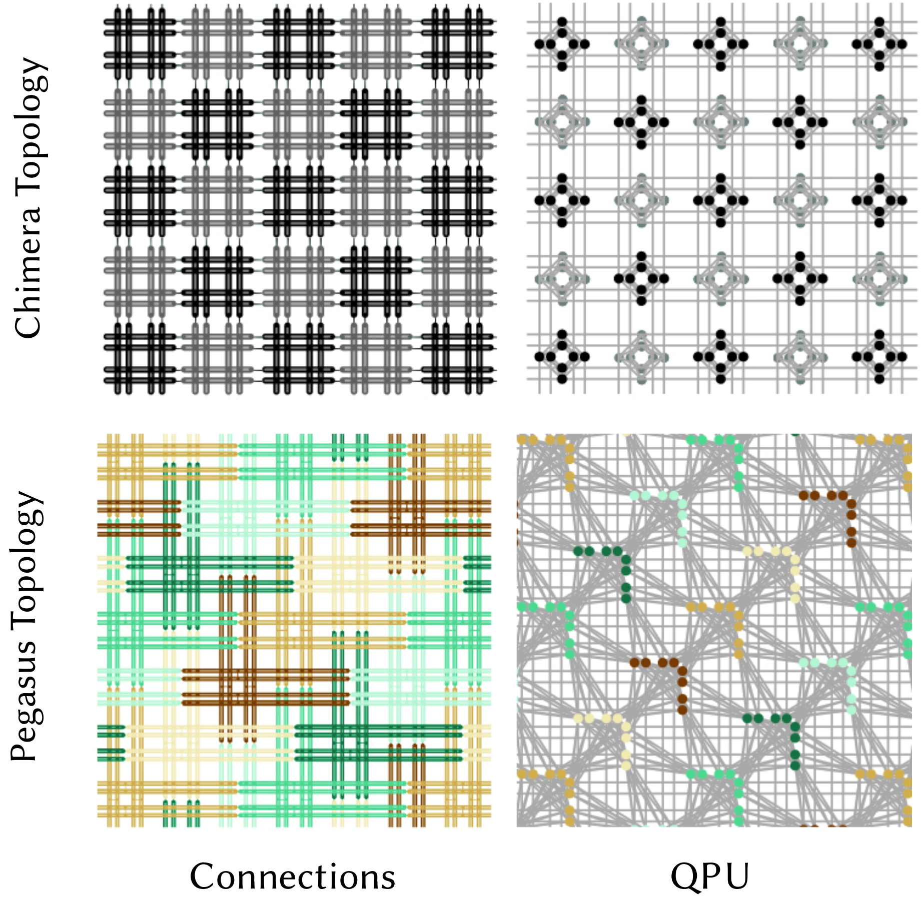

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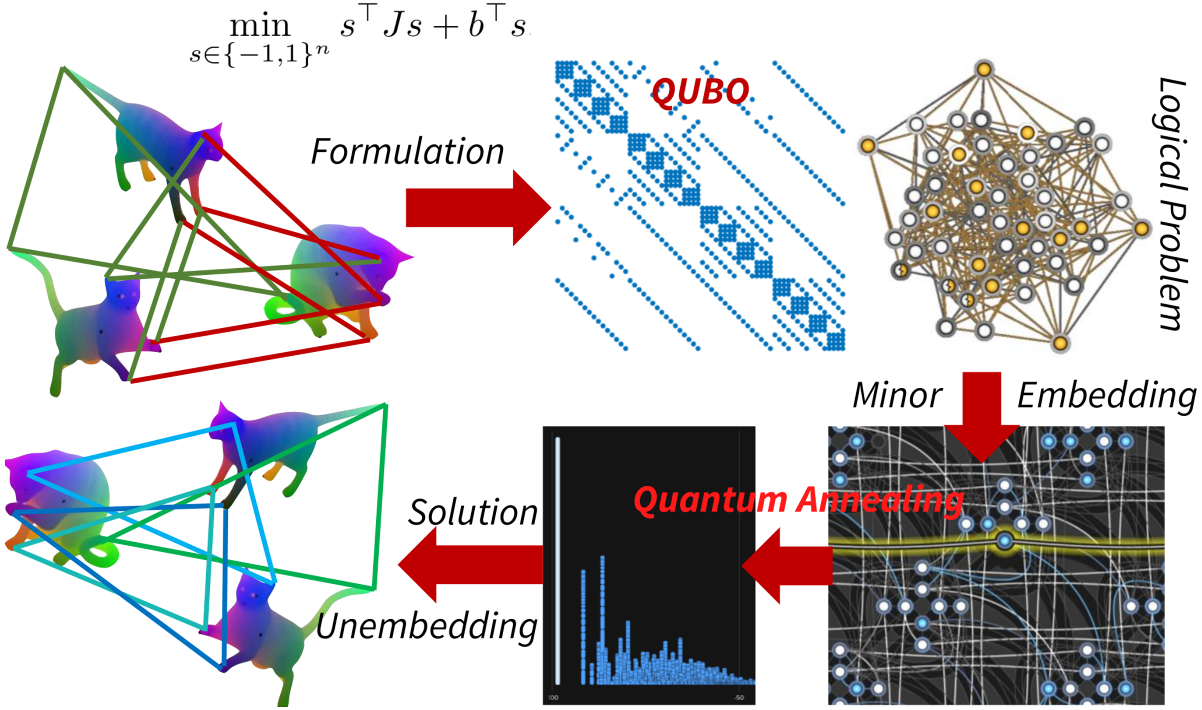

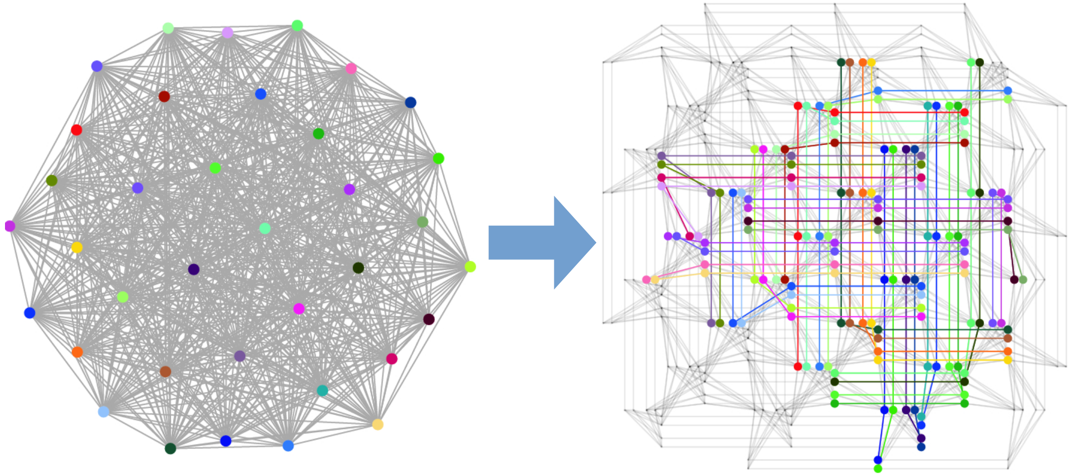

Fully connected graph with 36 logical qubits

Minor embedding

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Quantum Computer Vision

3-Satisfiability	3D image tomography
Bayesian inference in imaging	Binary matrix factorization
Budget pacing in auctions	Calibrating transmissions
Capacitated vehicle routing	Chemical structure analysis
Designing metamaterials	Display advertisement optimization
Donor-patient matching	Election modeling
Factoring	Factory vehicle scheduling
Fault diagnosis	Fault tree analysis
Financial stress analysis	Flight gate assignment
IMRT beamlet optimization	Job-shop scheduling
Linear least squares	List order optimization
Detecting LHC particle collisions	MIMO processing in radio networks
Minimizing polynomials	Modeling molecular dynamics
Model predictive control	Modeling terrorist networks
Number partitioning	Peptide design
Phylogenetics	Portfolio optimization
Satellite scheduling	Simulating KT phase transitions
Simulating material structures	Simulating atomic magnetometers
Simulating quantum lattice transitions	Stock market forecasting
Telecommunications network design	Topological data analysis
Traffic flow optimization	Traffic signal optimization
Tsunami evacuation routing	Waste collection optimization
ML: accelerating deep learning	ML: classification in cancer research
ML: classification in DNA analysis	ML: quantum boosting
ML: training neural networks	ML: reinforcement learning
ML: unsupervised learning	

D-Wave. Practical Quantum Computing: An Update. White Paper, 2020.

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Computer Vision and Graphics

Transformation estimation (2D, 3D)
Point set alignment
Graph matching
Permutation synchronisation
Non-rigid shape alignment
Motion segmentation
...

?

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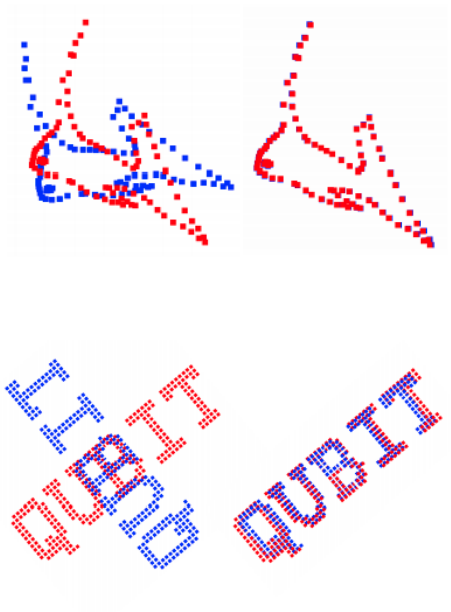
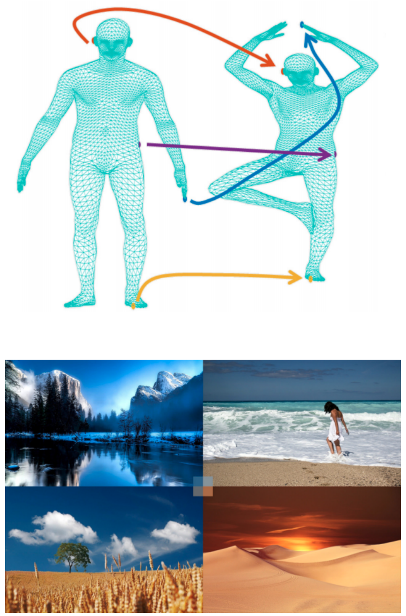
D-Wave. Practical Quantum Computing: An Update. White Paper, 2020.

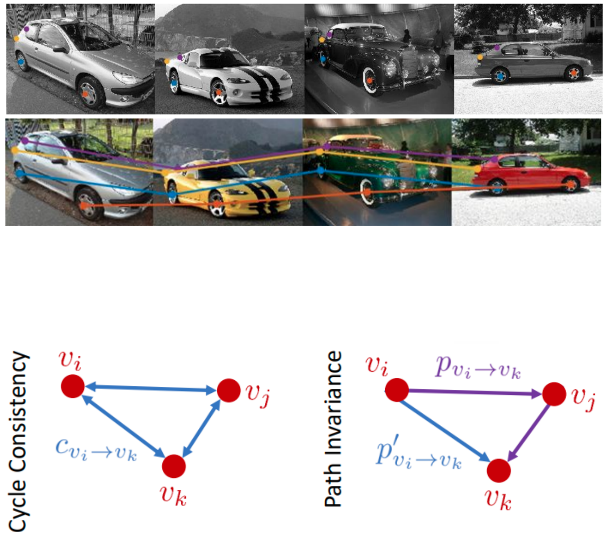
Quantum Computer Vision



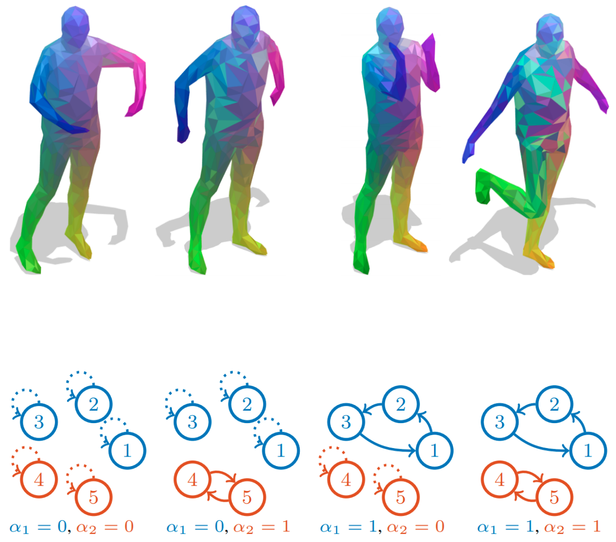
Point set registration, CVPR 2020



Graph matching, 3DV 2020



Permutation synchronisation, CVPR 2021



Non-rigid shape alignment, ICCV 2021

Golyanik and Theobalt. A Quantum Computational Approach to Correspondence Problems on Point Sets. *CVPR*, 2020.
Seelbach Benkner, Golyanik, Theobalt, Moeller. Adiabatic Quantum Graph Matching with Permutation Matrix Constraints. *3DV*, 2020.
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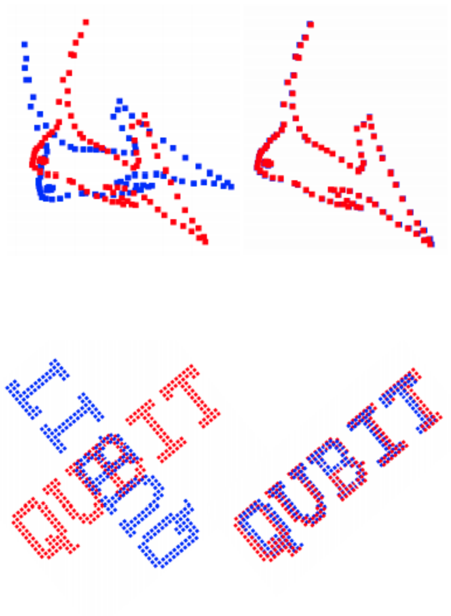
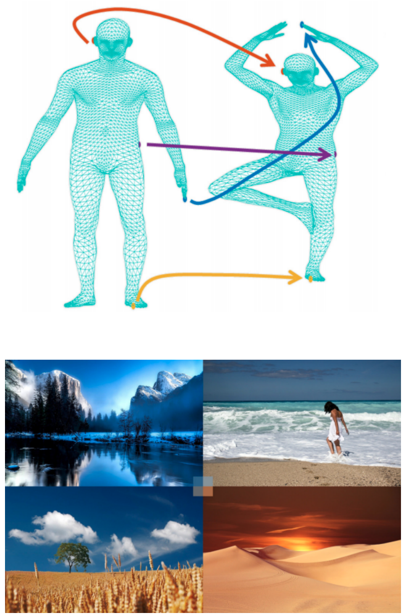
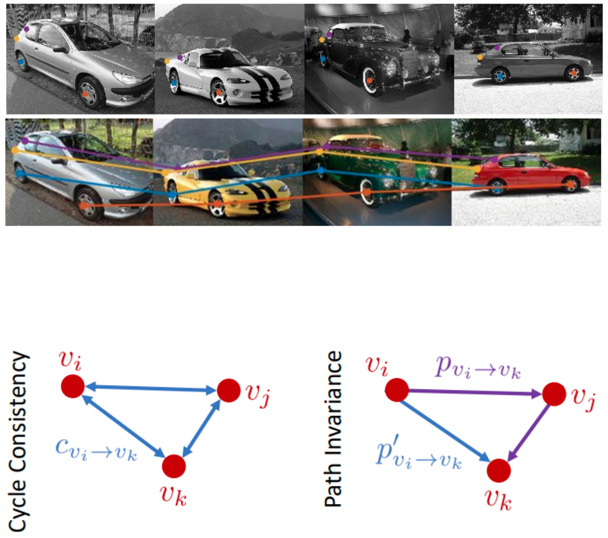
Quantum Computer Vision



Point set registration, CVPR 2020



Graph matching, 3DV 2020



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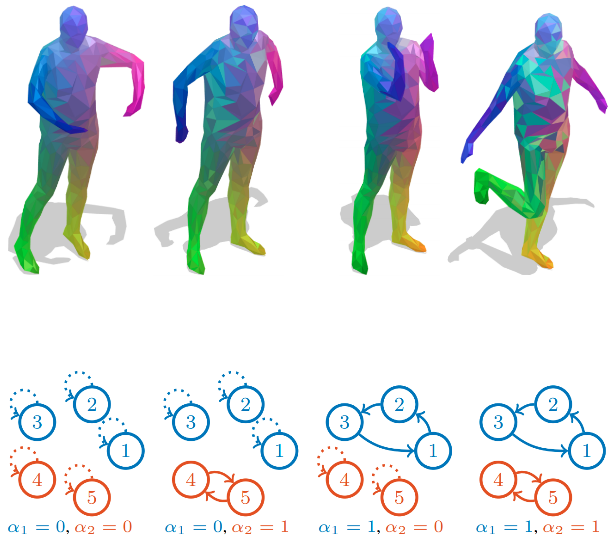

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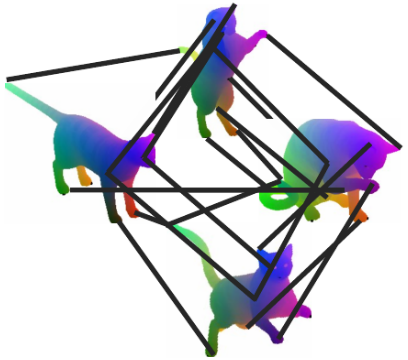
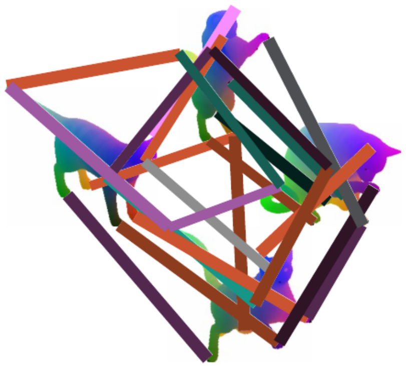
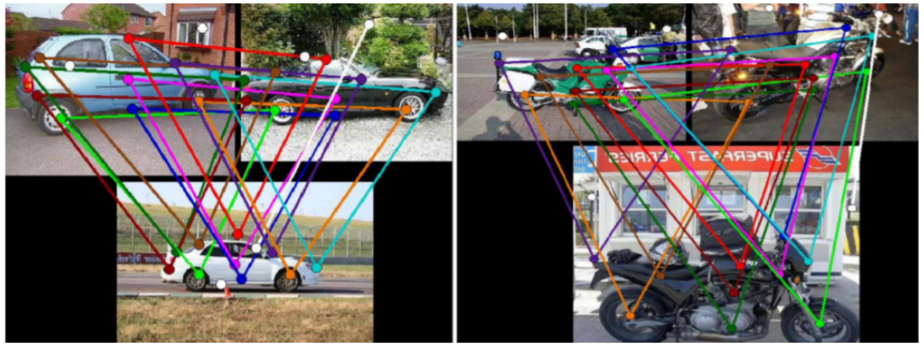
Quantum Permutation Synchronisation

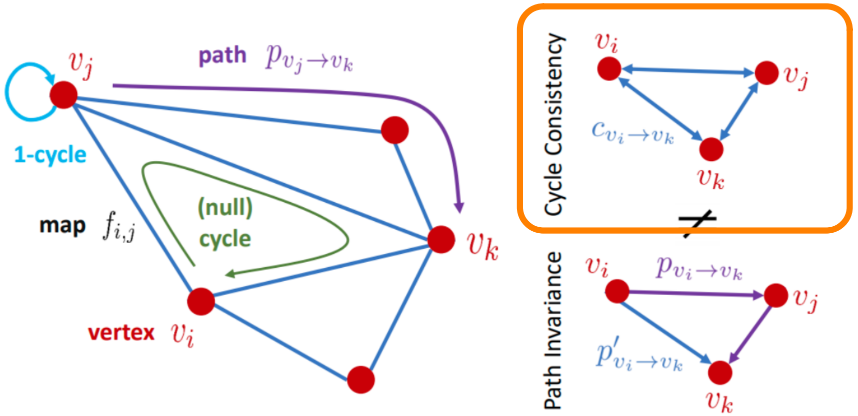


correct mistakes





2D or 3D inputs



$$\underset{\mathbf{x} \in \mathcal{B}^n}{\arg\min}\ \mathbf{x}^\top \mathbf{Q} \mathbf{x} + \mathbf{s}^\top \mathbf{x}$$

Binary Variables

Quadratic Term

Linear Term

Birdal,* Golyanik*, Theobalt, Guibas. Quantum Permutation Synchronization. *CVPR*, 2021.
*equal contribution

Quantum Permutation Synchronisation

$$\underset{\{\mathbf{X}_i \in \mathcal{P}_n\}}{\arg\min} \sum_{(i,j)\in\mathcal{E}} \|\mathbf{P}_{ij} - \mathbf{X}_i\mathbf{X}_j^\top\|_{\mathrm{F}}^2 = \underset{\{\mathbf{X}_i \in \mathcal{P}_n\}}{\arg\min}\ \mathbf{x}^\top\mathbf{Q}'\mathbf{x},$$

$$\mathbf{Q}' = -\begin{bmatrix} \mathbf{I}\otimes\mathbf{P}_{11} & \mathbf{I}\otimes\mathbf{P}_{12} & \cdots & \mathbf{I}\otimes\mathbf{P}_{1m} \\ \mathbf{I}\otimes\mathbf{P}_{21} & \mathbf{I}\otimes\mathbf{P}_{22} & \cdots & \mathbf{I}\otimes\mathbf{P}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{I}\otimes\mathbf{P}_{m1} & \mathbf{I}\otimes\mathbf{P}_{m2} & \cdots & \mathbf{I}\otimes\mathbf{P}_{mm} \end{bmatrix}.$$

Birdal,* Golyanik*, Theobalt, Guibas. Quantum Permutation Synchronization. *CVPR*, 2021.
*equal contribution

Quantum Permutation Synchronisation

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$$\arg\min_{\{\mathbf{X}_i \in \mathcal{P}_n\}} \sum_{(i,j)\in\mathcal{E}} \|\mathbf{P}_{ij} - \mathbf{X}_i\mathbf{X}_j^\top\|_{\mathrm{F}}^2 = \arg\min_{\{\mathbf{X}_i \in \mathcal{P}_n\}} \mathbf{x}^\top\mathbf{Q}'\mathbf{x},$$

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$$\arg\min_{\mathbf{x}\in\mathcal{B}} \mathbf{x}^\top\mathbf{Q}'\mathbf{x} \quad \textit{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{b}$$

is turned into

$$\arg\min_{\mathbf{x}\in\mathcal{B}} \mathbf{x}^\top\mathbf{Q}\mathbf{x} + \mathbf{s}^\top\mathbf{x},$$

where $\boxed{\mathbf{Q} = \mathbf{Q}' + \lambda\mathbf{A}^\top\mathbf{A}}$ *and* $\mathbf{s} = \boxed{-2\lambda\mathbf{A}^\top\mathbf{b}.}$

Birdal,* Golyanik*, Theobalt, Guibas. Quantum Permutation Synchronization. *CVPR*, 2021.
*equal contribution

Quantum Permutation Synchronisation

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$$\mathbf{A}_i = \begin{bmatrix} \mathbf{I}\otimes\mathbf{1}^\top \\ \mathbf{1}^\top\otimes\mathbf{I} \end{bmatrix} \qquad \mathbf{b}_i = \mathbf{1}$$

$$\Rightarrow$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & & & \\ & \mathbf{A}_2 & & \\ & & \ddots & \\ & & & \mathbf{A}_n \end{bmatrix} \qquad \mathbf{b} = \mathbf{1}$$

Birdal,* Golyanik*, Theobalt, Guibas. Quantum Permutation Synchronization. *CVPR*, 2021.
*equal contribution

Quantum Permutation Synchronisation

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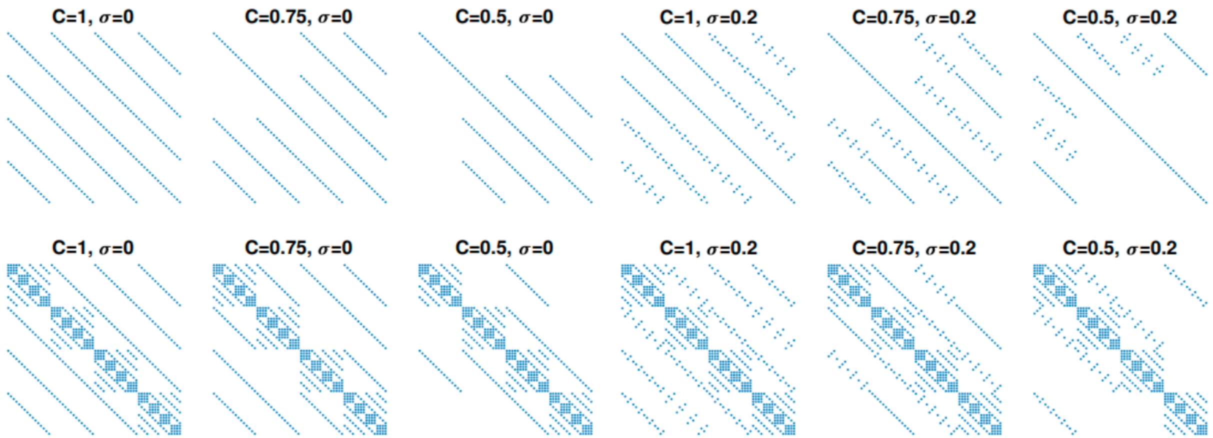

Exemplary $\mathbf{Q}$

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$$\Rightarrow$$

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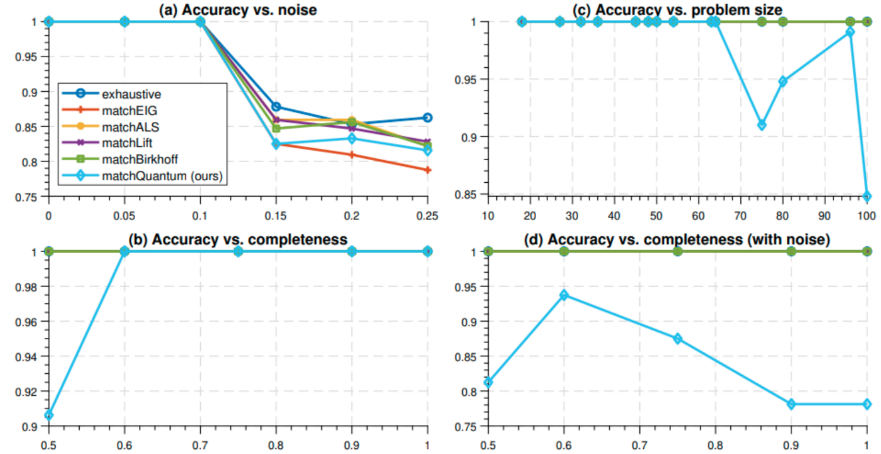
Birdal,* Golyanik*, Theobalt, Guibas. Quantum Permutation Synchronization. *CVPR*, 2021.
*equal contribution

Quantum Permutation Synchronisation

	Car	Duck	Motorbike	Winebottle	Average
Exhaustive	**0.84** ± **0.104**	**0.91** ± **0.115**	**0.82** ± **0.10**	**0.95** ± **0.096**	**0.88** ± **0.104**
EIG	0.81 ± 0.083	0.86 ± 0.102	0.77 ± 0.059	0.87 ± 0.107	0.83 ± 0.088
ALS	**0.84** ± **0.095**	0.90 ± 0.102	0.81 ± 0.078	0.94 ± 0.092	0.87 ± 0.092
LIFT	**0.84** ± **0.102**	0.90 ± 0.103	0.81 ± 0.078	0.94 ± 0.092	0.87 ± 0.094
Birkhoff	**0.84** ± **0.094**	0.90 ± 0.107	0.81 ± 0.079	0.94 ± 0.093	0.87 ± 0.093
D-Wave(Ours)	**0.84** ± **0.104**	0.90 ± 0.104	0.81 ± 0.080	0.93 ± 0.095	0.87 ± 0.096

Average Errors



Evaluations on the synthetic dataset (4 views and 4 points)

Birdal,* Golyanik*, Theobalt, Guibas. Quantum Permutation Synchronization. *CVPR*, 2021.
*equal contribution

Quantum Permutation Synchronisation

	Car	Duck	Motorbike	Winebottle	Average
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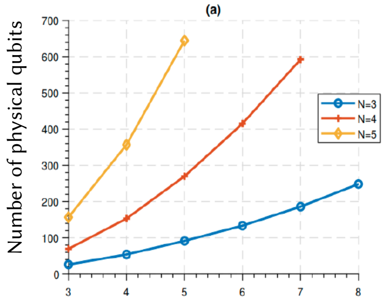
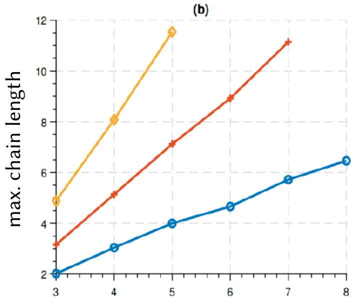
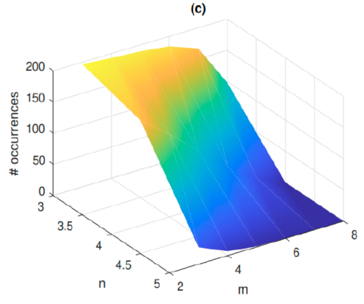
Average Errors



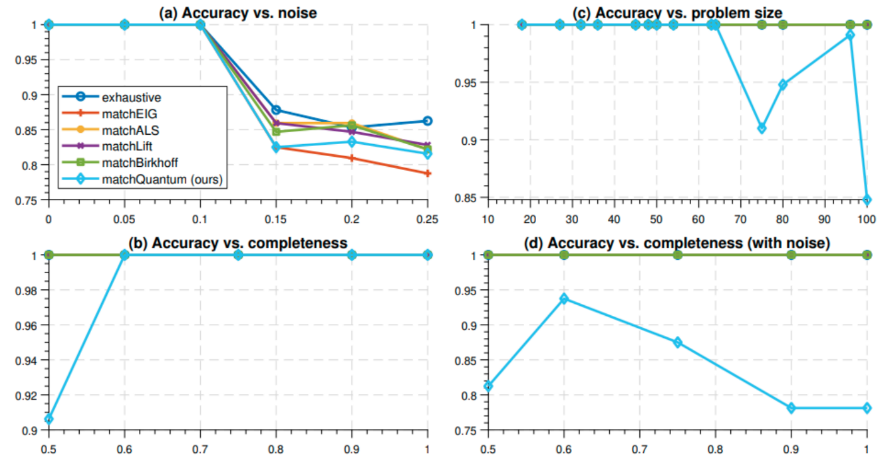
number of views



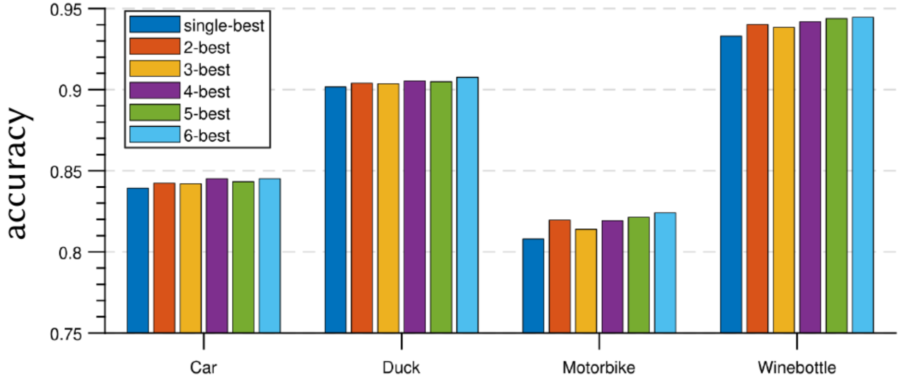
number of views



Average number of measured optimal solutions



Evaluations on the synthetic dataset (4 views and 4 points)



Bit corrections using multiple measurements of different energies

Birdal,* Golyanik*, Theobalt, Guibas. Quantum Permutation Synchronization. *CVPR*, 2021.
*equal contribution

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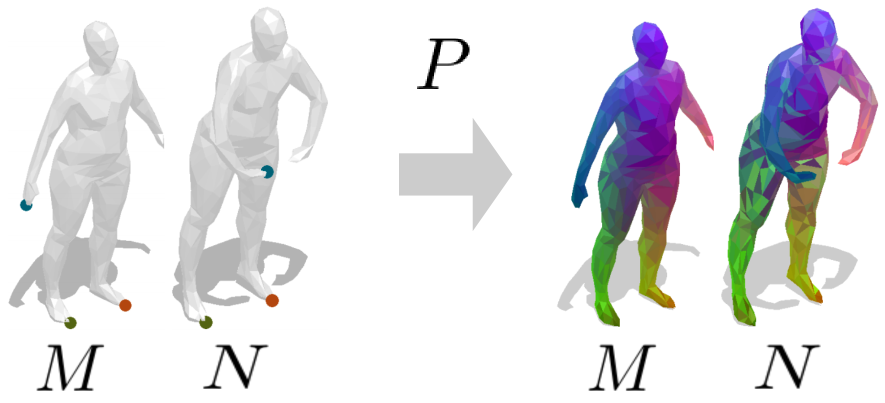
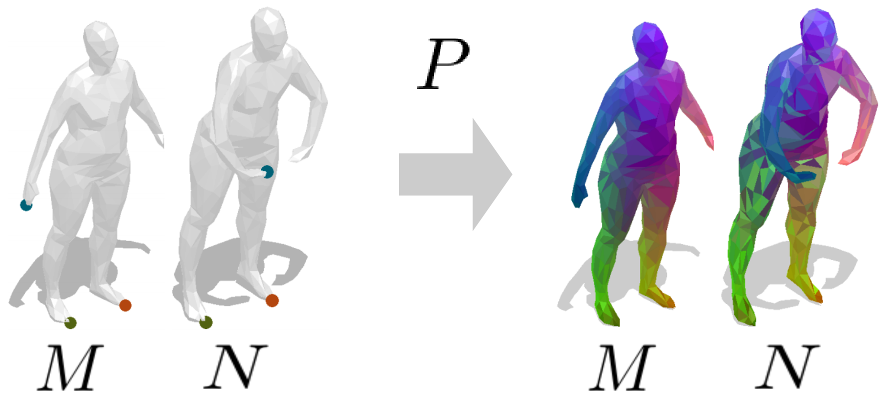
Q-Match: Iterative Shape Alignment



Seelbach Benkner, Lähner, Golyanik, Wunderlich, Theobalt, Moeller. Q-Match: Iterative Shape Matching via Quantum Annealing. *ICCV*, 2021.

Q-Match: Iterative Shape Alignment



$$\min_{X \in \mathbb{P}_n} \; E(X) := \mathbf{x}^{\mathrm{T}} W \mathbf{x}$$

$$\mathbf{x} \;=\; \mathrm{vec}(X) \qquad W \in \mathbb{R}^{n^2 \times n^2}$$

$$\mathbb{P} \;\subset\; \{0,1\}^{n\times n} \quad \text{(permutation matrix)}$$

$$\mathbb{P}_n \;=\; \{X \;\in\; \{0,1\}^{n\times n} \mid \textstyle\sum_i X_{ij} \;=\; 1,\; \sum_j X_{ij} \;=\; 1 \;\forall i,j\}.$$

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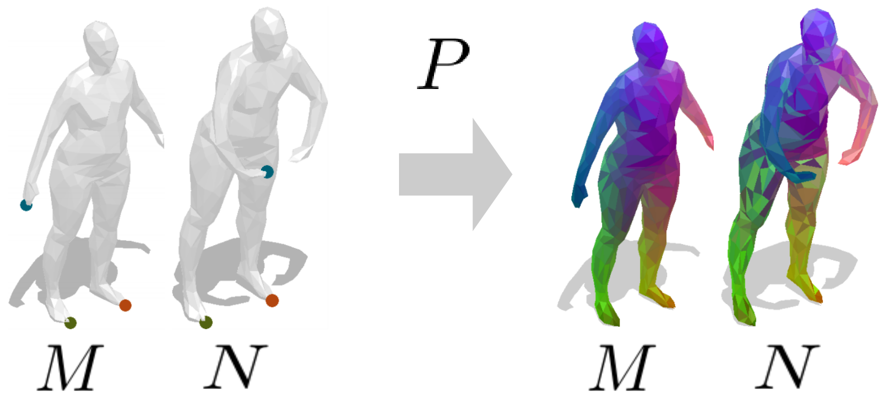

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$$\min_{X \in \mathbb{P}_n} E(X) := \mathbf{x}^{\mathrm{T}} W \mathbf{x}$$

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$$\mathbb{P}_n = \{X \in \{0,1\}^{n \times n} \mid \textstyle\sum_i X_{ij} = 1,\ \sum_j X_{ij} = 1\ \forall i,j\}.$$

k-cycles:

1 → 4 → 6 → 8 → 3 → 7 → 1

six-cycle

2, 5

fixed points

Disjoint permutations commute:

1 → 2 → 4 → 3 → 1

four-cycle

5

fixed point

6 ⇄ 7

two-cycle

Any X can be written as $X = \prod_{i=0}^{N} c_i$,
i.e., a product of 2-cycles
(or, generally, disjoint *k-cycles*).

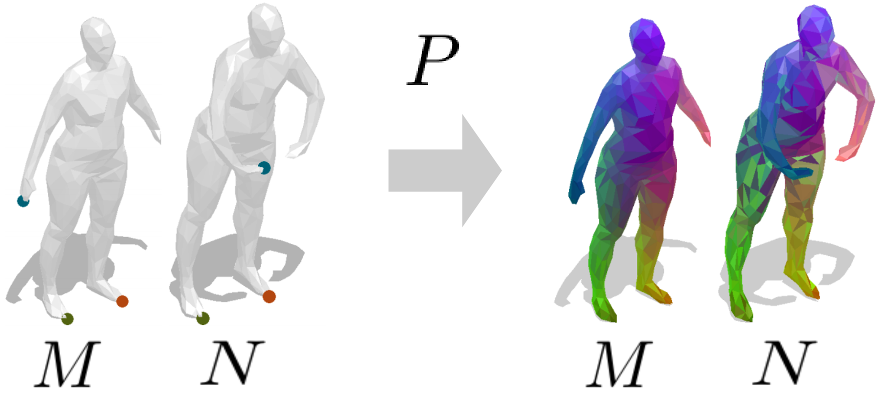
Seelbach Benkner, Lähner, Golyanik, Wunderlich, Theobalt, Moeller. Q-Match: Iterative Shape Matching via Quantum Annealing. *ICCV*, 2021.

Q-Match: Iterative Shape Alignment

Given: 3D shapes M and N, both discretised with n vertices.

$$W_{i \cdot n+k, j \cdot n+l} = |d^g_M(i,j) - d^g_N(k,l)|$$

Find: optimal P



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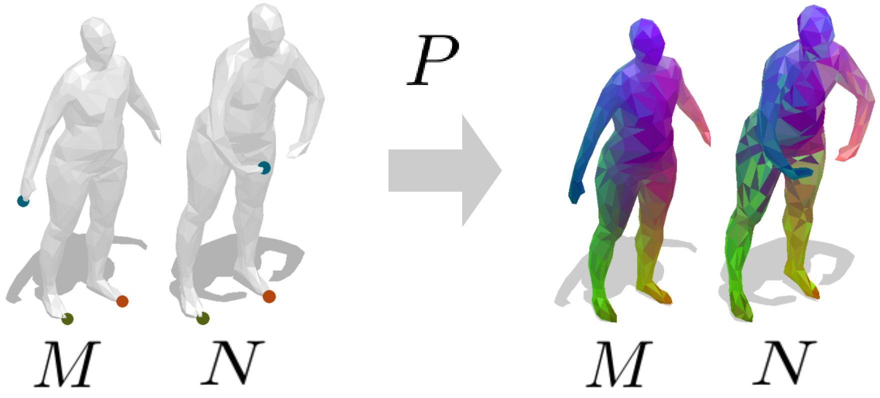

Want to solve but cannot:

$$\min_{X \in \mathbb{P}_n} E(X) := \mathbf{x}^{\mathrm{T}} W \mathbf{x}$$

$$W_{i\cdot n+k, j\cdot n+l} = |d^g_M(i,j) - d^g_N(k,l)|$$

Instead solve

$$\underset{\{P \in \mathbb{P}_n | \exists \alpha \in \{0,1\}^m : P = \left(\prod_i c_i^{\alpha_i}\right) P_0\}}{\arg\min} E(P)$$

$$C = \{c_1, \ldots, c_m\}$$

Seelbach Benkner, Lähner, Golyanik, Wunderlich, Theobalt, Moeller. Q-Match: Iterative Shape Matching via Quantum Annealing. *ICCV*, 2021.

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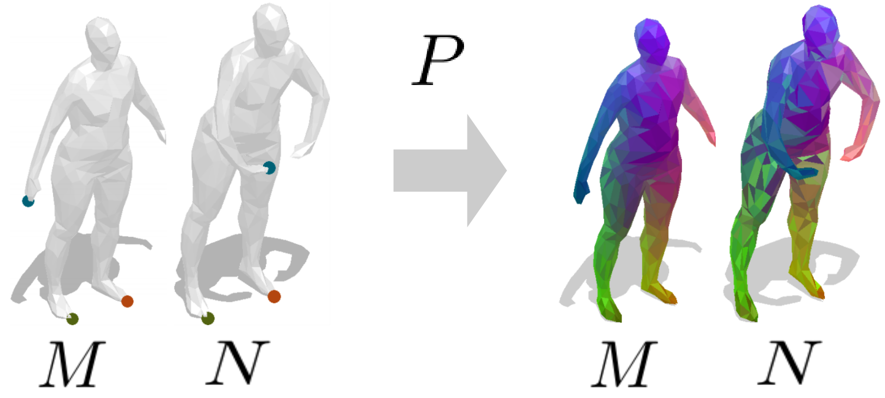

Want to solve but cannot:

$$\min_{X\in\mathbb{P}_n} E(X) := \mathbf{x}^\mathrm{T} W \mathbf{x}$$

$$W_{i\cdot n+k,j\cdot n+l} = |d^g_M(i,j) - d^g_N(k,l)|$$

... leading to

$$\min_{\alpha\in\{0,1\}^m} \alpha^\top \tilde{W}\alpha \qquad \tilde{W}_{ij} = \begin{cases} E(C_i, C_j) & \text{if } i \neq j, \\ E(C_i, C_i) + E(C_i, P_0) + E(P_0, C_j) & \text{otherwise.} \end{cases}$$

not submodular

Instead solve

$$\underset{\{P\in\mathbb{P}_n \mid \exists \alpha\in\{0,1\}^m:\ P=\left(\prod_i c_i^{\alpha_i}\right)P_0\}}{\arg\min} E(P)$$

$$C = \{c_1, ..., c_m\}$$

$$E(Q,R) = \mathrm{vec}(Q)^T W \mathrm{vec}(R)$$

$$P(\alpha) = P_0 + \sum_{i=1}^m \alpha_i \underbrace{(c_i - I)P_0}_{C_i}$$

Seelbach Benkner, Lähner, Golyanik, Wunderlich, Theobalt, Moeller. Q-Match: Iterative Shape Matching via Quantum Annealing. *ICCV*, 2021.

Q-Match: Iterative Shape Alignment

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Assume $C = \{c_1, ..., c_m\}$ is a set of disjoint cycles.

Consider
$$\underset{\{P \in \mathbb{P}_n \mid \exists \boxed{\alpha \in \{0,1\}^m}:\ P = \left(\prod_i c_i^{\alpha_i}\right) \boxed{P_0}\}}{\arg\min} E(P)$$

binary vector parametrising P

initial permutation

$$\begin{pmatrix} 1-\alpha_1 & 0 & \alpha_1 & 0 & 0 \\ \alpha_1 & 1-\alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_1 & 1-\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & 1-\alpha_2 & \alpha_2 \\ 0 & 0 & 0 & \alpha_2 & 1-\alpha_2 \end{pmatrix}$$

Parametrisation of all combinations with two binary variables

Seelbach Benkner, Lähner, Golyanik, Wunderlich, Theobalt, Moeller. Q-Match: Iterative Shape Matching via Quantum Annealing. *ICCV*, 2021.

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Q-Match: Iterative Shape Alignment

Assume $C = \{c_1, ..., c_m\}$ is a set of disjoint cycles.

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binary vector parametrising P

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Parametrisation of all combinations with two binary variables

---

$\min_{X \in \mathbb{P}_n} E(X) := \mathbf{x}^\mathrm{T} W \mathbf{x}$ **Initial QAP formulation;** cannot be solved on QPU

$\min_{\alpha \in \{0,1\}^m} \alpha^\top \tilde{W} \alpha$ **QUBO formulation** based on cyclic alpha-expansion; can be solved on QPU

W initial matrix of costs; large; cannot be precomputed and stored; its entries are computed on demand in each iteration

$\tilde{W}$ matrix of QUBO costs; requires known W

W_s a $k^2 \times k^2$ reduction of W based on *k* worst matches

Seelbach Benkner, Lähner, Golyanik, Wunderlich, Theobalt, Moeller. Q-Match: Iterative Shape Matching via Quantum Annealing. *ICCV*, 2021.

Q-Match: Iterative Shape Alignment

Initialise P_0 via descriptor-based similarity

repeat until converged

obtain I_M and I_N and choose from them a set of *k* random and disjoint 2-cycles

construct a submatrix of worst matches W_s

repeat until every 2-cycle occurred

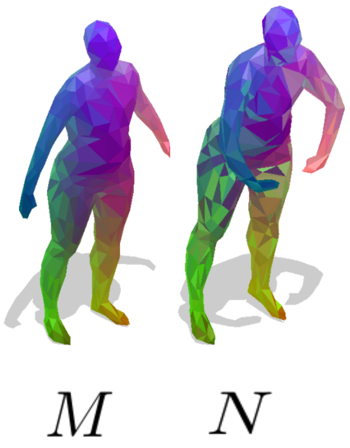
choose a random set of 2-cycles

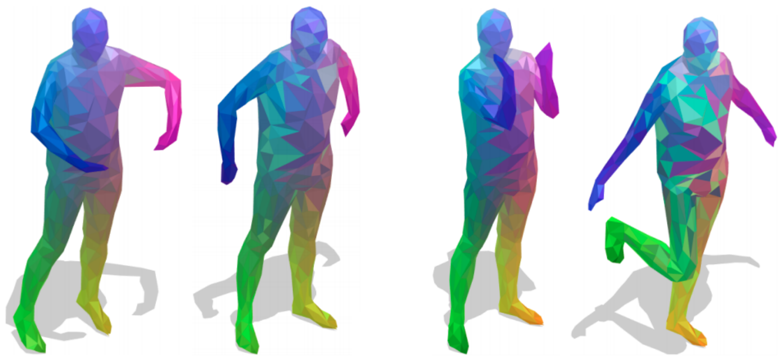
calculate $\tilde{W}$ and solve $\min_{\alpha \in \{0,1\}^m} \alpha^\top \tilde{W} \alpha$ on QPU

$P_i = \left(\prod_j c_j^{\alpha_j}\right) P_{i-1}$

apply the obtained permutation to worst matches

NP-hard; decides to apply c_i or not





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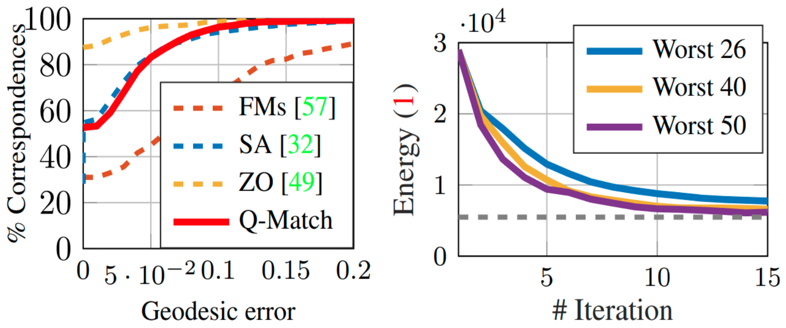
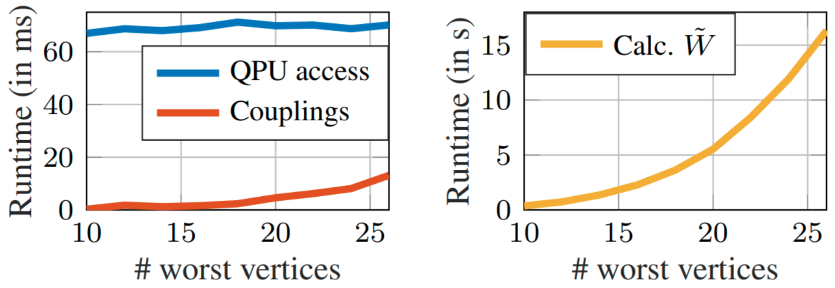
Q-Match: Iterative Shape Alignment

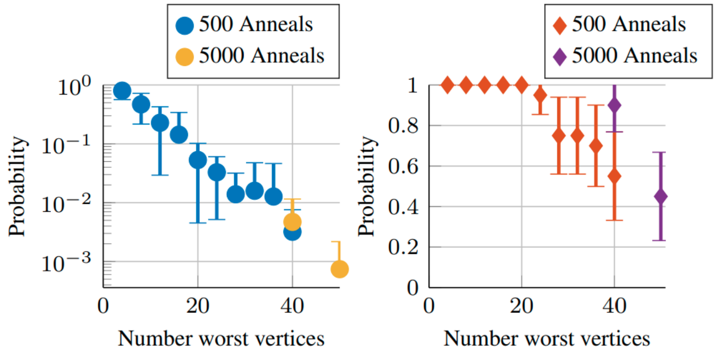
Retrieving the optimum



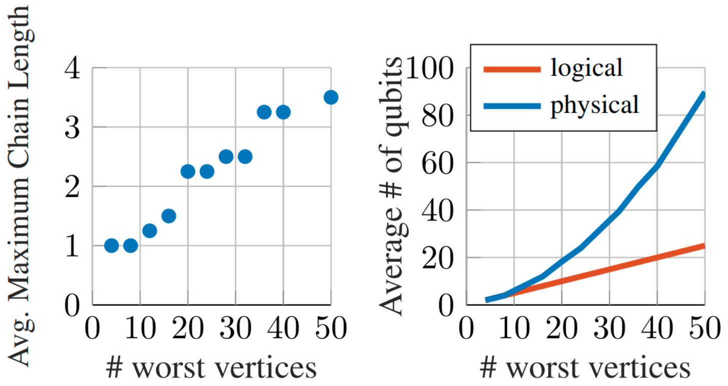
Cumulative error and convergence (FAUST)



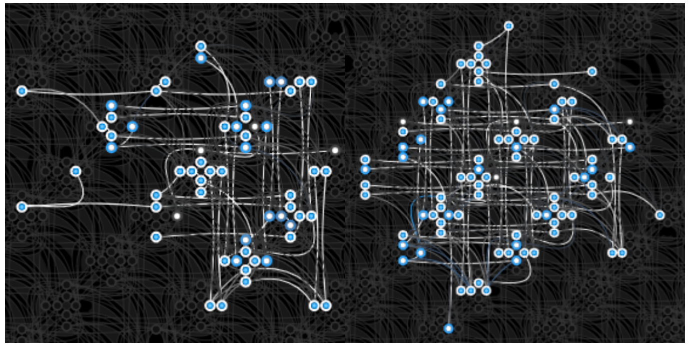
Runtime analysis



Success probabilities



Minor embedding functions



Exemplary minor embeddings
(using 40 and 50 worst vertices)

[32] Holzschuh *et al.* Simulated annealing for 3d shape correspondence. *3DV*, 2020.
[49] Melzi *et al.* Zoomout: Spectral upsampling for efficient shape correspondence. *ACM ToG*, 2019.
[57] Ovsjanikov *et al.* Functional maps: a flexible representation of maps between shapes. *ACM ToG*, 2012.
[67] Seelbacneh Benkner *et al.* Adiabatic Quantum Graph Matching with Permutation Matrix Constraints. *3DV*, 2020.

Seelbach Benkner, Lähner, Golyanik, Wunderlich, Theobalt, Moeller. Q-Match: Iterative Shape Matching via Quantum Annealing. *ICCV*, 2021.

Conclusions and Open Challenges

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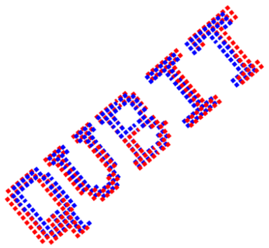
- Problem types: matching and synchronisation
- Input data: pairwise permutations, point sets, graphs, meshes
- Competitive results for small problems (if solved as a single QUBO sampling) and real-world problems (an iterative CPU-QPU policy)
- Iterative policies with CPU-QPU tasks are promising
- Runtime of the state preparation (QUBO) is not negligible
- The proposed algorithms will improve with hardware improvements (2000Q *vs* Advantage system1.1)



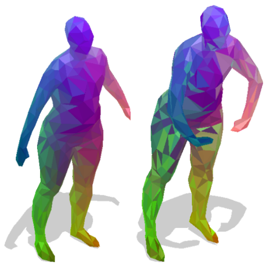


Image sources: Agarwal *et al.*, ICCV 2009; Meo *et al.*, Sensors, 2012; Gao *et al.*, CVPR, 2021.

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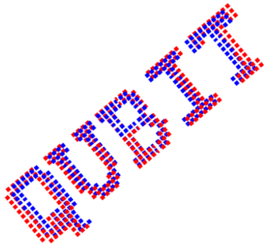
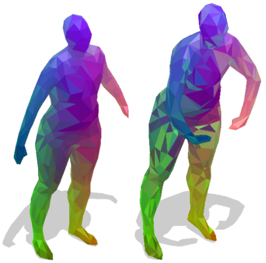
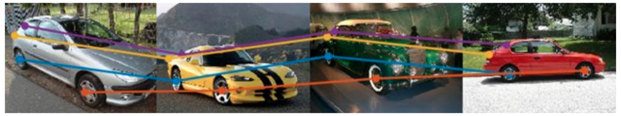
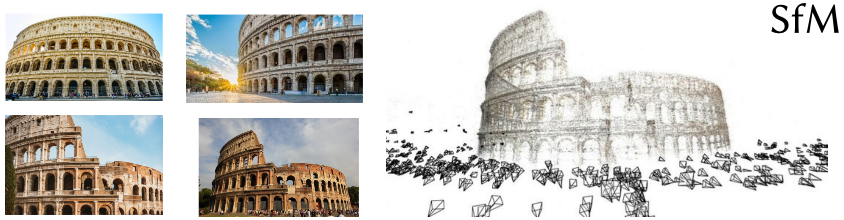








SfM

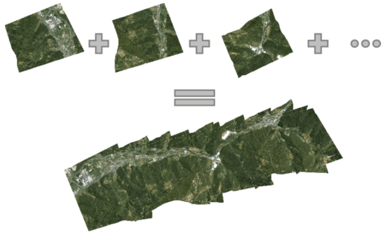
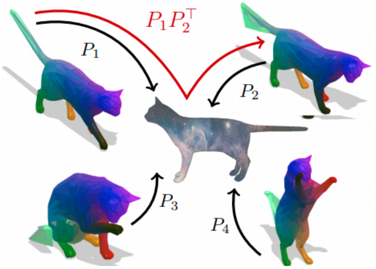


Image mosaicking



Multi-shape matching

- Many signs that quantum computing technology will continue developing next decades
- We are expecting to see works on quantum computer vision from more and more research groups
- Related research fields: quantum machine learning, circuit-based QC algorithms

Questions?

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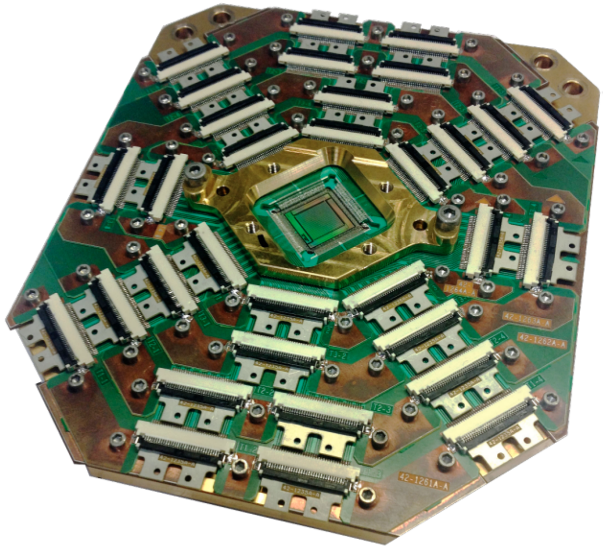

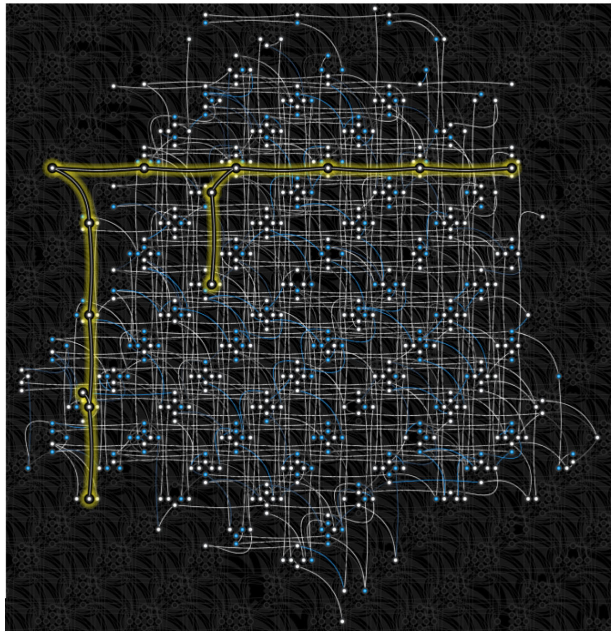

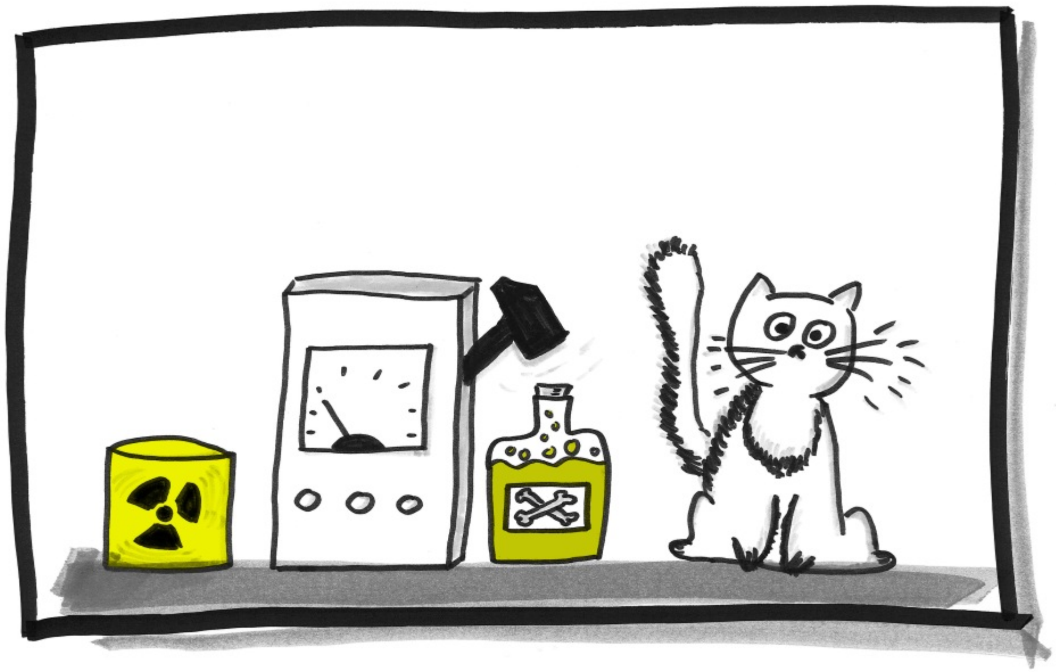

Image of Advantage sys. 1.1 (qubit and QPU); D-Wave Systems
https://www.leifiphysik.de/atomphysik/quantenmech-atommodell/versuche/schroedingers-katze-ein-gedankenexperiment

Thank You!