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Quantum Multi-Model Fitting

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JUNE 18-22, 2023 CVPR VANCOUVER, CANADA

Motivations? Contributions!

(?) The combinatorial nature of Multi Model Fitting: MMF deals with finding the best parametric models explaining a set of noisy and outlier-corrupted observations. Since many explanations exist, **MMF is inherently combinatorial**.

(!) QuMF: by providing the first QUBO formulation for MMF, we are able to solve MMF problems with Adiabatic Quantum Computers.

(?) Scalability with Adiabatic Quantum Computers: AQCs are a class of quantum computers capable of solving QUBO problems in constant time. They support little resource allocation, and hence **cannot scale to large problems**.

(!) DeQuMF: we introduce an iterative shrinkage of the search space that enables large resource allocation on current AQCs.

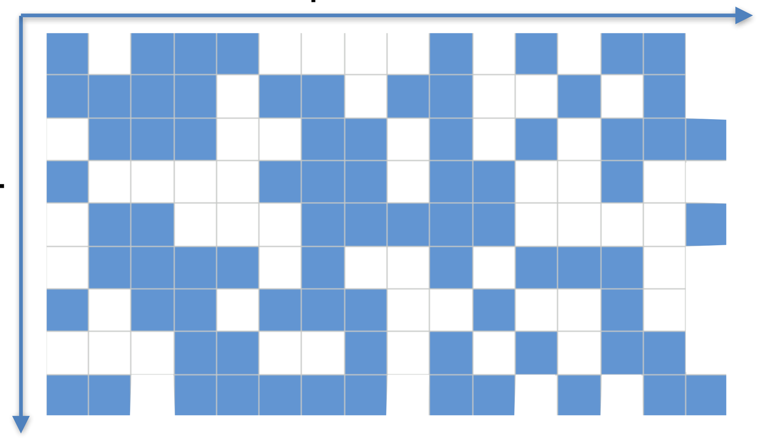
MMF as Disjoint Set Cover

In a *preference matrix*, the columns are the models' **consensus sets:**

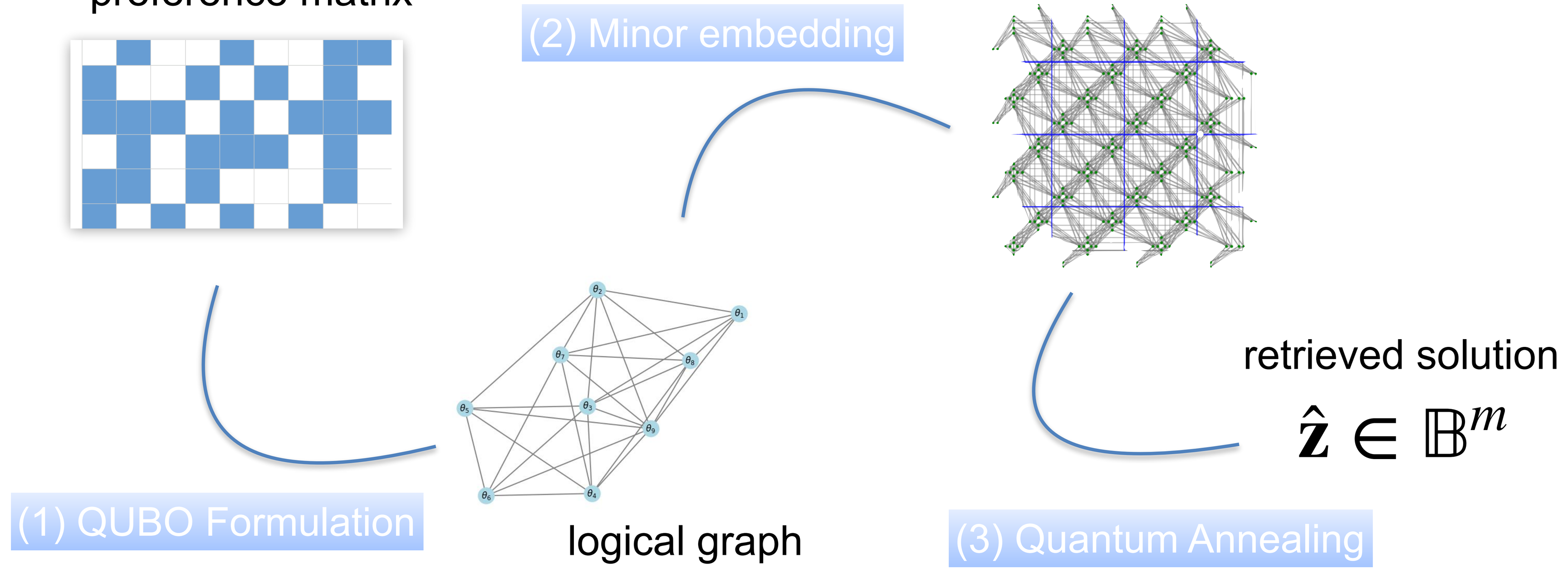
$$P[i,j] = \begin{cases} 1 & \text{if } error(x_i, \theta_j) < \epsilon \\ 0 & \text{otherwise} \end{cases}$$

Disjoint Set-Cover

$$\min_{\mathbf{z} \in \mathbb{B}^m} \mathbf{1}_m^{\mathsf{T}} \mathbf{z} \quad \text{s.t. } P\mathbf{z} = \mathbf{1}_n$$



Nice, but… how does Quantum Annealing work?



QuMF: Quantum Multi-Model Fitting

QuMF (**Qu**antum **M**ulti-Model **F**itting) addresses the combinatorial nature of Multi Model Fitting, by bringing a disjoint set-cover MMF formulation into QUBO form and employing Quantum Annealing

Soft Disjoint Set-Cover

$$\min_{\mathbf{z} \in \mathbb{B}^m} \mathbf{1}_m^{\mathsf{T}} \mathbf{z} + \lambda ||P\mathbf{z} - \mathbf{1}_n||_2^2$$

$\longrightarrow$

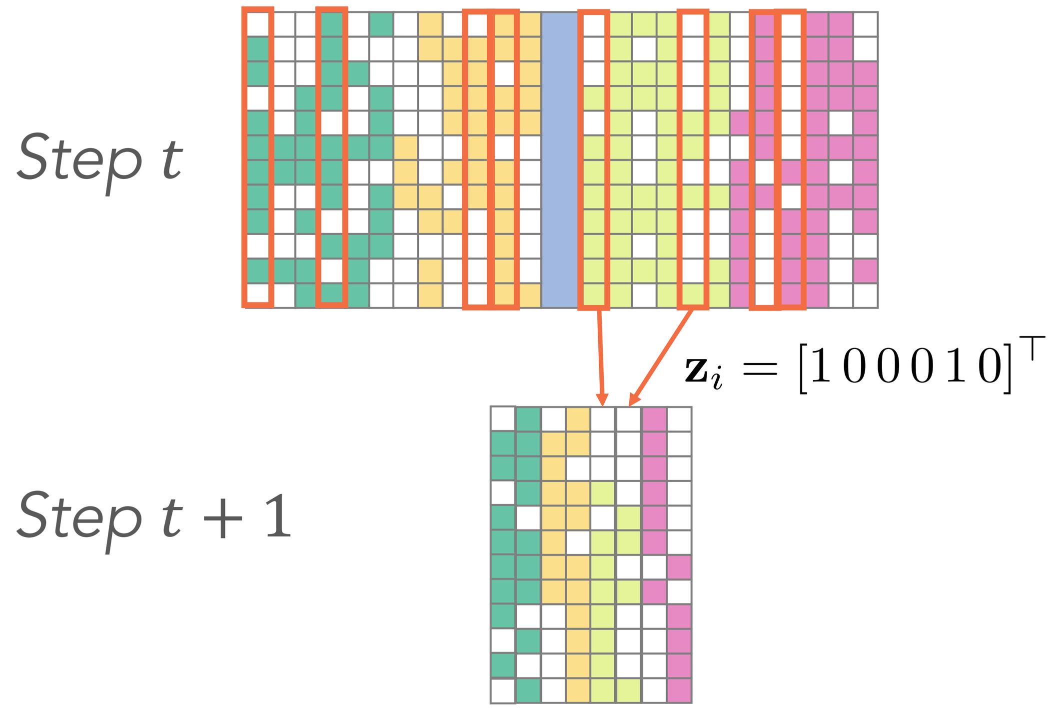
Quadratic Unconstrained Binary Optimisation

$$\min_{\mathbf{z} \in \mathbb{B}^m} \lambda \mathbf{z}^{\mathsf{T}} (P^{\mathsf{T}} P) \mathbf{z} + (\mathbf{1}_m - 2\lambda P^{\mathsf{T}} \mathbf{1}_n)^{\mathsf{T}} \mathbf{z}$$

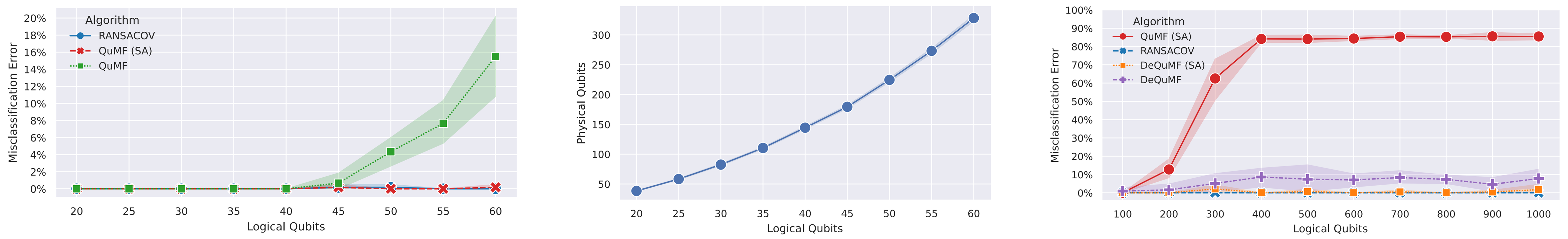
DeQuMF: Decomposed QuMF

DeQuMF (**De**composed **Qu**antum **M**ulti-Model **F**itting) tackles **scalability**.

It divides the QUBO into subproblems, and iteratively shrinks the search space by independently solving them with QuMF



Synthetic Experiments



DeQuMF enables tackling large-scale problems also on Quantum Hardware.

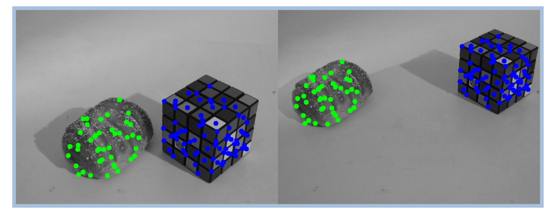
Real Experiments

Misclassification Error [%] on the Hopkins and York Datasets

		Multi-X	J-Linkage	T-Linkage	RPA	RanSaCov	QuMF (SA)	DeQuMF	DeQuMF (SA)
Traffic3	*mean*	0.32	1.58	0.48	**0.19**	0.35	5.14	8.74	0.55
	median	**0**	0.34	0.19	**0**	0.19	2.85	7.50	0.28
Traffic2	*mean*	**0.09**	1.75	1.31	0.14	0.54	6.04	-	0.10
	median	**0**	**0**	**0**	**0**	**0**	3.17	-	**0**
York	*mean*	-	2.85	1.44	1.08	**0.19**	12.29	-	0.74
	median	-	1.80	**0**	**0**	**0**	3.78	-	**0**

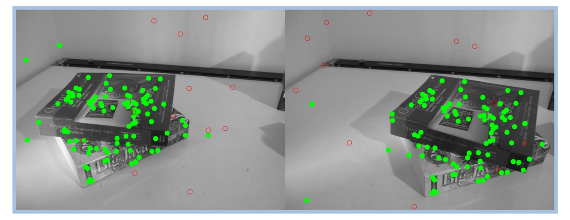
Misclassification Error [%] on the Adelaide Dataset

	RanSaCov [1]	QuMF (SA)	DeQuMF	DeQuMF (SA)
mean	9.79	3.85	16.22	**0.77**
median	7.97	3.54	11.0	**0.18**



DeQuMF for Single Model Fitting

Algorithm	Outlier ratio 10%	20%	Full sequence
QuMF (SA)	7.22	11.34	13.23
DeQuMF	**2.41**	10.53	16.17
DeQuMF (SA)	6.26	**8.28**	**10.83**
HQC-RF [3]	3.71	37.0	45.84



Finally, we study how DeQuMF behaves in a scenario falling completely out of its design: Single Model Fitting with outliers. Although not displaying SotA behaviour, DeQuMF exhibits robustness to increasing outlier ratio.

References & Acknowledgements

[1] RanSaCov: Magri, Luca, and Andrea Fusiello. "Multiple model fitting as a set coverage problem." CVPR 2016.
[2] Visualisation of the QPU Topology graph from the official DWave Documentation.
[3] HQC-RF: Doan, Anh-Dzung, et al. "A hybrid quantum-classical algorithm for robust fitting." CVPR 2022.
[4] Arrigoni, Federica, et al. "Quantum Motion Segmentation." ECCV 2022

This paper is supported by FAIR (Future Artificial Intelligence Research) project, funded by the NextGenerationEU program within the PNRR-PE-AI scheme (M4C2, Investment 1.3, Line on Artificial Intelligence). This work was partially supported by the PRIN project LEGO-AI (Prot. 2020TA3K9N) and it was carried out in the Vision and Learning joint laboratory of FBK and UNITN.

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