Quantum Motion Segmentation

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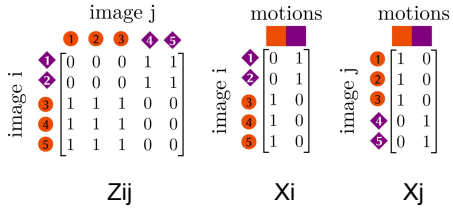
Overview and Contributions

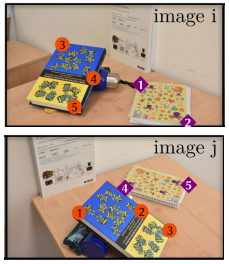
- Task of motion segmentation (MS) is to *classify points in multiple images into different motions*

- Applications of motion segmentation: multi-body SfM [45], motion estimation [51]
- MS is an NP-Hard problem
- **Contributions** of this paper:
  - QuMoSeg: A new MS approach employing AQC
  - A new Q-MSEG dataset for MS
- QuMoSeg reaches SotA accuracy on a wide range of problems (QA and SA versions)

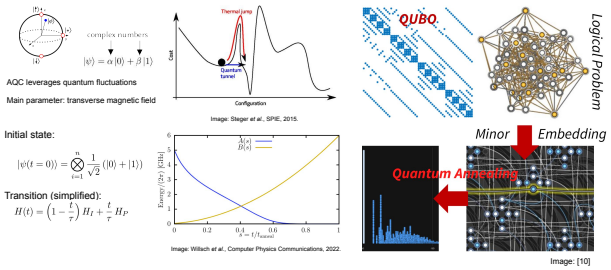
Motion Segmentation

- **Goal**: identify independent motions in multiple images
- **Assumption:** two-frame matches as input, which are used to recover two-frame segmentations Zij via multi-model fitting [40]
- **Task**: recover unknown absolute segmentations X1, …, Xn such that they are consistent with the known two-frame segmentations Zij, namely: $Z_{ij} = X_i X_j^\top$





Adiabatic Quantum Computing



- It optimizes objectives over **binary variables** and obtains globally-optimal or low-energy solutions with high probabilities, leveraging quantum mechanics.
- The objective must be expressed as a **QUBO** (quadratic unconstrained binary optimization)

$$\min_{\mathbf{y}\in\mathcal{B}^k} \mathbf{y}^\top Q\mathbf{y} + \mathbf{s}^\top\mathbf{y}$$

- Linear constraints are treated as soft ones

$$\min_{\mathbf{y}\in\mathcal{B}^k} \mathbf{y}^\top Q\mathbf{y} + \mathbf{s}^\top\mathbf{y} + \sum_i \lambda_i \,||A_i\mathbf{y} - \mathbf{b_i}||^2$$

Our Approach

Motion segmentation via synchronization (**combinatorial objective**)

$$\min_{X_1,\dots X_n} \sum_{(i,j)\in\mathcal{E}} ||Z_{ij} - X_i X_j^\top||_F^2,$$

$$\text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i\mathbf{1}_d = \mathbf{1}_{pi} \quad \forall i = 1,\dots,n$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \dots \\ X_n \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & Z_{12} \dots & Z_{1n} \\ Z_{21} & 0 \dots & Z_{2n} \\ \dots & & \dots \\ Z_{n1} & Z_{n2} \dots & 0 \end{bmatrix}$$

binary variables ⇔ Each point belongs to one motion only

QuMoSeg-v1 (no additional assumptions, dense matrix)

$$\max_{X_1,\dots X_n} \sum_{(i,j)\in\mathcal{E}} \text{trace}(X_i^\top(2Z_{ij} - \mathbf{1}_{p_i\times p_j})X_j) \quad \text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i\mathbf{1}_d = \mathbf{1}_{pi} \quad \forall i = 1,\dots,n$$

$$\Leftrightarrow \quad \max_X \text{vec}(X)^\top(I_{d\times d} \otimes (2Z - \mathbf{1}_{p\times p}))\,\text{vec}(X) \quad \text{s.t. } \text{vec}(X) \in \mathcal{B}^{dp}, \quad X\mathbf{1}_d = \mathbf{1}_p.$$

we know the number of points per motion in every image

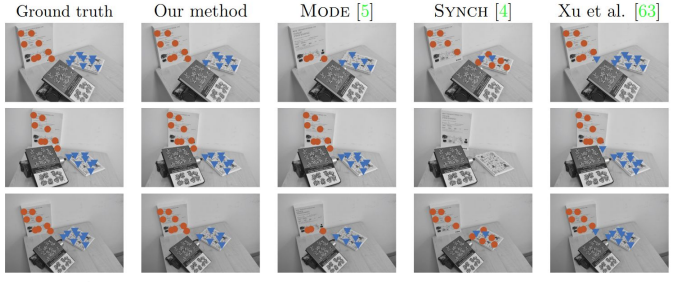
QuMoSeg-v2 (additional assumptions, sparse matrix)

$$\max_{X_1,\dots X_n} \sum_{(i,j)\in\mathcal{E}} \text{trace}(X_i^\top Z_{ij} X_j), \quad \text{s.t. } \text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i\mathbf{1}_d = \mathbf{1}_{pi}, \quad \mathbf{1}_{p_i}^\top X_i = \mathbf{m}_i^\top \quad \forall i = 1,\dots,n.$$

$$\Leftrightarrow \quad \max_X \text{vec}(X)^\top(I_{d\times d} \otimes Z)\,\text{vec}(X), \quad \text{s.t. } \text{vec}(X) \in \mathcal{B}^{dp}, \quad X\mathbf{1}_d = \mathbf{1}_p, \quad KX = M$$

additional constraints

Experimental Results



Qualitative results on Q-MSEG dataset with 96 qubits. Each color (symbol) represents a distinct planar motion.

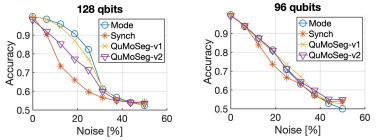
Q-MSEG

# Qubits:	96	102	120	126	128	136	160	168	180	190	200	216	220	243
Xu et al. [63]	0.89	0.89	0.94	0.75	0.96	0.97	0.86	0.86	0.97	0.88	0.96	0.77	0.83	0.74
MODE [5]	0.93	0.93	0.96	0.93	**0.97**	0.97	**0.98**	0.99	**0.98**	**0.99**	**0.99**	0.93	1	**0.94**
SYNCH [4]	0.93	0.94	0.95	0.95	0.84	0.92	0.97	1	0.89	0.95	0.90	**0.94**	0.99	0.92
QUMOSEG-V1	0.97	0.97	0.97	0.96	0.95	**0.98**	**0.98**	0.99	**0.98**	**0.99**	**0.99**	0.64	–	–
QUMOSEG-V2	0.96	0.97	0.95	0.94	0.89	0.89	0.88	0.85	0.74	0.75	0.79	0.59	0.75	0.58
QUMOSEG-V1, SA	0.97	0.97	0.97	0.96	0.95	**0.98**	**0.98**	1	**0.98**	**0.99**	**0.99**	0.68	0.98	0.72
QUMOSEG-V2, SA	**0.98**	**0.99**	**0.99**	1	0.96	**0.98**	**0.98**	1	0.94	0.97	**0.99**	0.80	1	0.59

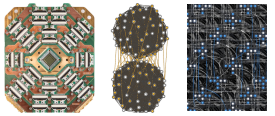
Hopkins

# Qubits:	120	126	132	138	144	156	162	168	174	180	186	192	198	204	210	216	222	228	234	240
Xu et al. [63]	0.80	0.78	0.81	0.79	0.83	0.81	0.84	0.81	0.85	0.89	0.88	0.94	0.96	0.96	0.97	1	0.98	1	**0.99**	1
MODE [5]	0.89	0.91	0.90	0.93	0.92	0.94	0.95	0.95	0.96	0.95	0.97	0.98	0.98	0.98	0.98	0.99	0.98	0.98	**0.99**	0.99
SYNCH [4]	0.87	0.93	0.95	0.96	**0.99**	0.96	**0.99**	**0.99**	0.96	0.99	1	1	0.99	1	1	0.70	0.97	1	**0.99**	0.65
QUMOSEG-V1	0.92	0.89	0.93	0.93	0.93	0.93	0.95	0.94	0.96	0.95	0.96	0.97	0.96	-	-	-	-	-	-	-
QUMOSEG-V2	0.91	0.92	0.91	0.92	0.94	0.89	0.91	0.89	0.90	0.88	0.88	0.89	0.88	0.89	0.88	-	-	-	-	-
QUMOSEG-V1, SA	0.93	0.90	0.92	0.94	0.93	0.94	0.95	0.96	0.96	0.96	0.98	0.98	0.98	0.99	0.99	0.98	0.98	**0.99**	0.99	
QUMOSEG-V2, SA	**0.96**	**0.97**	**0.98**	**0.98**	**0.99**	**0.99**	**0.99**	0.97	**0.99**	1	1	1	1	1	1	**0.99**	1	**0.99**	1	

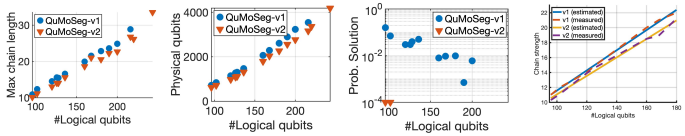
Increasing Noise Levels



Minor Embeddings



Execution on D-Wave Quantum Annealer



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